## COMPUTER SCIENCE TRIPOS Part II - 2018 - Paper 8

## 3 Computer Systems Modelling (RJG)

This question deals with stochastic processes $\{N(t), t \geq 0\}$ where $N(t)$ represents the number of events in the time interval $[0, t]$.
(a) (i) Define a Poisson process $\{N(t), t \geq 0\}$ of rate $\lambda>0$.
(ii) Show that $N(t) \sim \operatorname{Pois}(\lambda t)$ for each fixed $t>0$. You may use the result that $\lim _{n \rightarrow \infty}(1-x / n)^{n}=e^{-x}$ without proof.
(iii) Let $X_{1}$ be the time of the first event of the Poisson process $N(t)$. Show that $X_{1} \sim \operatorname{Exp}(\lambda)$.
[2 marks]
(iv) Now given that $N(t)=1$ derive the distribution of the time of the single event in $[0, t]$.
[4 marks]
(b) Suppose that events of a Poisson process of rate $\lambda$ are independently selected at random with probability $p>0$. Show that the process of selected events is also a Poisson process and establish its rate.
(c) Describe how your result from part (b) can be used to simulate a nonhomogeneous Poisson process whose rate function $\lambda(t)$ is such that $\lambda(t) \leq \lambda^{*}$ for all $t \geq 0$.

