## COMPUTER SCIENCE TRIPOS Part II – 2018 – Paper 8

## 3 Computer Systems Modelling (RJG)

This question deals with stochastic processes  $\{N(t), t \ge 0\}$  where N(t) represents the number of events in the time interval [0, t].

- (a) (i) Define a Poisson process  $\{N(t), t \ge 0\}$  of rate  $\lambda > 0$ . [2 marks]
  - (*ii*) Show that  $N(t) \sim \text{Pois}(\lambda t)$  for each fixed t > 0. You may use the result that  $\lim_{n\to\infty} (1-x/n)^n = e^{-x}$  without proof. [4 marks]
  - (*iii*) Let  $X_1$  be the time of the first event of the Poisson process N(t). Show that  $X_1 \sim \text{Exp}(\lambda)$ . [2 marks]
  - (*iv*) Now given that N(t) = 1 derive the distribution of the time of the single event in [0, t]. [4 marks]
- (b) Suppose that events of a Poisson process of rate  $\lambda$  are independently selected at random with probability p > 0. Show that the process of selected events is also a Poisson process and establish its rate. [2 marks]
- (c) Describe how your result from part (b) can be used to simulate a nonhomogeneous Poisson process whose rate function  $\lambda(t)$  is such that  $\lambda(t) \leq \lambda^*$ for all  $t \geq 0$ . [6 marks]