5 Computation Theory (AD)

(*a*) Give a precise definition of the class of *partial recursive functions.* [3 marks]

(*b*) We can associate with each natural number $i \in \mathbb{N}$ the partial recursive function $f_i : \mathbb{N} \rightharpoonup \mathbb{N}$ computed by the register machine coded by the number i. Explain why

(*i*) for every partial recursive function $f : \mathbb{N} \rightharpoonup \mathbb{N}$, there is an i such that $f = f_i$; and

(*ii*) the partial function $g : \mathbb{N} \times \mathbb{N} \rightharpoonup \mathbb{N}$ given by $g(i, n) = f_i(n)$ is computable.

[8 marks]

(*c*) Show that the total function $T : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ given by:

$$T(i, n) = \begin{cases} 1 & \text{if } f_i(n)\downarrow \\ 0 & \text{otherwise} \end{cases}$$

is uncomputable. Here f_i refers to the partial function associated with $i \in \mathbb{N}$ as in (*b*). [9 marks]