COMPUTER SCIENCE TRIPOS Part IA – 2018 – Paper 2

10 Discrete Mathematics (IML)

(a) Let $\Sigma = \{a, b\}$ and let $\#_x(w)$ be the number of occurrences of the symbol $x \in \Sigma$ in the string w. For each of the following determine, with justification, whether or not the language is regular.

(i)
$$L_1 = \{ w \in \Sigma^* \mid \#_a(w) = \#_b(w) \}$$
 [3 marks]

- (*ii*) $L_2 = \{ w \in \Sigma^* \mid w \text{ has an equal number of occurrences of the substrings <math>ab$ and $ba \}$. [3 marks]
- (*iii*) L_3 inductively defined by the following axiom and rule:

$$\frac{u}{\epsilon} \qquad \frac{u}{aub} \qquad \text{for all } u \in \Sigma^*$$
 [3 marks]

(iv) L_4 inductively defined by the following axiom and rules:

$$\frac{u}{\epsilon} \qquad \frac{u}{aub} \qquad \frac{u}{au} \qquad \frac{u}{ub} \qquad \text{for all } u \in \Sigma^* \qquad [3 \text{ marks}]$$

(v)
$$L_5 = \{ w \in \Sigma^* \mid (\#_a(w) = 3i) \land (\#_b(w) = 7j) \text{ for some } i, j \in \mathbb{N} \}$$
 [3 marks]

- (b) Consider the set R of all regular expressions over the alphabet $\{a, b\}$.
 - (i) Give an alphabet sufficient to express any element of R. [2 marks]
 - (ii) State, giving reasons, whether R is a regular language. [3 marks]