## COMPUTER SCIENCE TRIPOS Part IA - 2018 - Paper 2

## 10 Discrete Mathematics (IML)

(a) Let $\Sigma=\{a, b\}$ and let $\#_{x}(w)$ be the number of occurrences of the symbol $x \in \Sigma$ in the string $w$. For each of the following determine, with justification, whether or not the language is regular.
(i) $L_{1}=\left\{w \in \Sigma^{*} \mid \#_{a}(w)=\#_{b}(w)\right\}$
(ii) $L_{2}=\left\{w \in \Sigma^{*} \mid w\right.$ has an equal number of occurrences of the substrings $a b$ and $b a\}$.
(iii) $L_{3}$ inductively defined by the following axiom and rule:

$$
\bar{\epsilon} \quad \frac{u}{a u b} \quad \text { for all } u \in \Sigma^{*}
$$

(iv) $L_{4}$ inductively defined by the following axiom and rules:

$$
\bar{\epsilon} \quad \frac{u}{a u b} \quad \frac{u}{a u} \quad \frac{u}{u b} \quad \text { for all } u \in \Sigma^{*} \quad \text { [3 marks] }
$$

(v) $L_{5}=\left\{w \in \Sigma^{*} \mid\left(\#_{a}(w)=3 i\right) \wedge\left(\#_{b}(w)=7 j\right)\right.$ for some $\left.i, j \in \mathbb{N}\right\} \quad[3$ marks]
(b) Consider the set $R$ of all regular expressions over the alphabet $\{a, b\}$.
(i) Give an alphabet sufficient to express any element of $R$.
(ii) State, giving reasons, whether $R$ is a regular language.

