

6 Denotational Semantics (MPF)

Consider the concocted language PCF_* obtained from the language PCF by extending it with:

- Types
 $\tau ::= \dots \mid \tau * \tau$
- Expressions
 $M ::= \dots \mid \mathbf{pair}(M, M) \mid \mathbf{left}(M) \mid \mathbf{right}(M)$
- Typing rules

$$\frac{\Gamma \vdash M_1 : \tau_1 \quad \Gamma \vdash M_2 : \tau_2}{\Gamma \vdash \mathbf{pair}(M_1, M_2) : \tau_1 * \tau_2} \quad \frac{\Gamma \vdash M : \tau_1 * \tau_2}{\Gamma \vdash \mathbf{left}(M) : \tau_1} \quad \frac{\Gamma \vdash M : \tau_1 * \tau_2}{\Gamma \vdash \mathbf{right}(M) : \tau_2}$$

- Values
 $V ::= \dots \mid \mathbf{pair}(V, V)$
- Operational semantics

$$\frac{M_1 \Downarrow_{\tau_1} V_1 \quad M_2 \Downarrow_{\tau_2} V_2}{\mathbf{pair}(M_1, M_2) \Downarrow_{\tau_1 * \tau_2} \mathbf{pair}(V_1, V_2)}$$

$$\frac{M \Downarrow_{\tau_1 * \tau_2} \mathbf{pair}(V_1, V_2)}{\mathbf{left}(M) \Downarrow_{\tau_1} V_1} \quad \frac{M \Downarrow_{\tau_1 * \tau_2} \mathbf{pair}(V_1, V_2)}{\mathbf{right}(M) \Downarrow_{\tau_2} V_2}$$

- (a) Give a denotational semantics for the above extension of PCF. [3 marks]
- (b) Show that the denotation of types are domains and that the denotation of terms are continuous functions. You may use any standard results provided that you state them clearly. [5 marks]
- (c) State the soundness property for a denotational semantics of PCF_* . [2 marks]
- (d) Show that your denotational semantics of PCF_* is sound. You may use any standard results provided that you state them clearly. [4 marks]
- (e) State the adequacy property for a denotational semantics of PCF_* . [2 marks]
- (f) Establish whether or not your denotational semantics of PCF_* is adequate. You may use any standard results provided that you state them clearly. [4 marks]