COMPUTER SCIENCE TRIPOS Part II – 2017 – Paper 8

2 Machine Learning and Bayesian Inference (SBH)

(a) Give a detailed definition of a *hidden Markov model* (HMM). Include in your answer a description of the assumptions made by this model.   [4 marks]

(b) Derive the *Viterbi algorithm* for computing, on the basis of some HMM, the most likely sequence of states to have produced a given sequence of observations.
[6 marks]

(c) An HMM has three states $\{s_1, s_2, s_3\}$ with the prior

$$\Pr(S_0 = s_1) = 0.3, \Pr(S_0 = s_2) = 0.3, \Pr(S_0 = s_3) = 0.4.$$

The transition model is

$$\mathbf{T} = \left(\begin{array}{ccc} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.4 & 0.5 \\ 0.6 & 0.1 & 0.3 \end{array} \right)$$

where $\mathbf{T}_{ij} = \Pr(S_{t+1} = s_j | S_t = s_i)$. Observations have values $\{e_1, e_2\}$, and the sensor model is

$$\Pr(E_t = e_1 | S_t = s_1) = 0.6, \Pr(E_t = e_1 | S_t = s_2) = 0.5, \Pr(E_t = e_1 | S_t = s_3) = 0.1.$$

You observe that at times 0, 1 and 2 the three corresponding observations are (e_1, e_2, e_1). Use the Viterbi algorithm to compute the most likely sequence (S_0, S_1, S_2) of states.   [10 marks]