10 Information Theory (JGD)

(*a*) An error-correcting (7/4) Hamming code combines four data bits b_3, b_5, b_6, b_7 with three error-correcting bits: $b_1 = b_3 \oplus b_5 \oplus b_7$, $b_2 = b_3 \oplus b_6 \oplus b_7$, and $b_4 = b_5 \oplus b_6 \oplus b_7$. The 7-bit block is then sent through a noisy channel, which corrupts one of the seven bits. The following incorrect bit pattern is received:

b_1	b_2	b_3	b_4	b_5	b_6	b_7
1	1	0	1	0	0	0

Evaluate three syndromes that are derived upon reception of the 7-bit block: $s_1 = b_1 \oplus b_3 \oplus b_5 \oplus b_7$, $s_2 = b_2 \oplus b_3 \oplus b_6 \oplus b_7$, $s_4 = b_4 \oplus b_5 \oplus b_6 \oplus b_7$, and provide the corrected 7-bit block that was the input to this noisy channel. [5 marks]

(*b*) Consider a noisy channel that suffers from additive noise which is not uniform in power across frequencies, but where instead the noise power attenuates exponentially with frequency ω. This channel has a signal-to-noise ratio $\text{SNR}(\omega)$ that therefore improves at higher frequencies, as: $\text{SNR}(\omega) = 2^\omega - 1$ over the possible range. Within some interval $[\omega_1, \omega_2]$ of the spectrum, with $\omega_2 > \omega_1 > 0$, what is the information capacity C of this noisy channel in bits/sec? [5 marks]

(*c*) Explain the Lempel-Ziv lossless compression algorithm, including its principle, its steps, its payload requirements, and its adaptive variants. [5 marks]

(*d*) The following traces show how an IrisCode bit sequence (bottom) is computed from a sample of human iris texture (top), for compact pattern encoding whose high entropy makes identification very reliable. Explain the rôles of Gabor wavelets and of Logan's Theorem in reference to these traces. [5 marks]