1 Complexity Theory (AD)

Let $S \subseteq \mathbb{N}$ be a set of numbers. We write bin$S$ for the set of binary strings (i.e. strings in $\{0,1\}^*$) x such that x is a binary representation of a number in S. We also write un$S$ for the set $\{a^k \mid k \in S\}$ where the notation a^k means the string consisting of k repetitions of the letter a.

(*a*) Suppose bin$S \in \mathsf{TIME}(2^{cn})$ for some constant c. Prove that un$S \in \mathsf{P}$.

[4 marks]

(*b*) Give definitions of the complexity classes L and NL. [4 marks]

(*c*) Prove that:

(*i*) bin$S \in \mathsf{SPACE}(n)$ if, and only if, un$S \in \mathsf{L}$; and

(*ii*) bin$S \in \mathsf{NSPACE}(n)$ if, and only if, un$S \in \mathsf{NL}$.

[6 marks]

(*d*) Recall that Reach is the problem of reachability in directed graphs. Using part (*c*) or otherwise, show that, if Reach were in L, it would follow that $\mathsf{SPACE}(n) = \mathsf{NSPACE}(n)$. State carefully any standard results that you use.

[6 marks]