COMPUTER SCIENCE TRIPOS Part IA – 2017 – Paper 1

10 Algorithms (DJW)

(*a*) Consider a directed acyclic graph with V vertices and E edges.

(*i*) What is meant by *a total order on the vertices consistent with the edges*? [2 marks]

(*ii*) Describe an $O(E+V)$ algorithm to compute such a total order. [3 marks]

(*b*) Consider a directed graph with non-negative edge costs and with a given start vertex s.

(*i*) Dijkstra's algorithm computes distances from s to every other vertex. Give psuedocode for Dijkstra's algorithm. [4 marks]

(*ii*) Dijkstra's algorithm can be implemented using a Fibonacci heap. State the complexity of using this implementation. Justify your answer carefully. [*Note:* Your answer should include mention of amortized costs.] [4 marks]

(*c*) Consider a directed acyclic graph with non-negative edge costs and with a given start vertex s.

(*i*) Devise an algorithm to compute distances from s in $O(E+V)$ time. Justify why your algorithm is correct. [4 marks]

(*ii*) Explain, with an example, why Dijkstra's algorithm might take $\Omega(V \log V)$ time. [3 marks]