5 Logic and Proof (LCP)

(*a*) Write brief notes on the use of clause methods to prove theorems. Include a description of an algorithm that can find a model of a set of clauses, if one exists. Illustrate your answer using the following example:

$$\{P, Q, \neg R\} \quad \{\neg P, R\} \quad \{\neg Q\} \quad \{P, R\}$$

[6 marks]

(*b*) For each of the following sets of clauses, either exhibit a model or show that none exists. Below, a and b are constants, while x, y and z are variables.

(*i*)

$$\{\neg P(x), Q(x, x)\}$$
$$\{\neg Q(x, y), \neg Q(y, x), R(x, y)\}$$
$$\{\neg R(x, y), \neg R(y, x)\}$$
$$\{P(a), P(b)\}$$

[7 marks]

(*ii*)

$$\{P(x), Q(x)\}$$
$$\{\neg P(x), Q(f(x))\}$$
$$\{P(x), \neg Q(f(x))\}$$
$$\{\neg P(x), \neg Q(x)\}$$

[7 marks]