2 Complexity Theory (AD)

The *Graph Isomorphism* problem is the problem of deciding, given two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, whether there is a bijection $\beta : V_1 \to V_2$ such that

$$(u, v) \in E_1 \quad \text{if, and only if,} \quad (\beta(u), \beta(v)) \in E_2,$$

for all $u, v \in V_1$.

The Graph Isomorphism problem is not known to be in P nor known to be NP-complete.

We define GI to be the set of all languages L which are *polynomial-time reducible* to Graph Isomorphism.

What can you conclude from the above definitions and information about the truth of the following statements? If the statement is true or false, justify your answer and if you cannot conclude anything about its truth, explain why that is so.

(*a*) Graph Isomorphism is in NP. [4 marks]

(*b*) Graph Isomorphism is in co-NP. [4 marks]

(*c*) GI $\subseteq$ NP. [3 marks]

(*d*) NP $\subseteq$ GI. [3 marks]

(*e*) P $\subseteq$ GI. [3 marks]

(*f*) GI $\subseteq$ P. [3 marks]