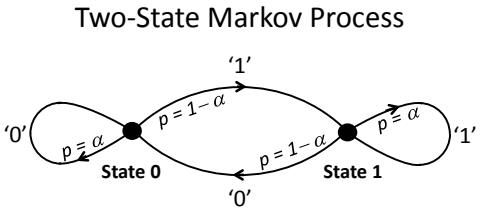
COMPUTER SCIENCE TRIPOS Part II – 2015 – Paper 7

9 Information Theory and Coding (JGD)

(*a*) A two-state Markov process may emit a '0' in State 0 or a '1' in State 1, each with probability α, and return to the same state; or with probability $1-\alpha$ it emits the other symbol and switches to the other state. Thus it tends to be "sticky" or oscillatory, two forms of predictability, depending on α.



(*i*) What are the state occupancy probabilities for $0 < \alpha < 1$? [2 marks]

(*ii*) What are the entropy of State 0, the entropy of State 1, and the overall entropy of this source? Express your answers in terms of α. [2 marks]

(*iii*) For what value(s) of α do both forms of predictability disappear? What then is the entropy of this source, in bits per emitted bit? [2 marks]

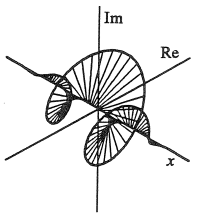
(*b*) Consider a binary symmetric channel with error probability p that any bit may be flipped. Two possible error-correcting coding schemes are available.

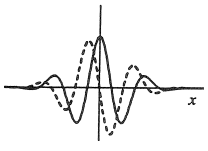
(*i*) Without any error-correcting coding scheme in place, state all the conditions that would maximise the channel capacity. Include conditions on the error probability p and also on the probability distribution of the binary source input symbols. [2 marks]

(*ii*) If a (7/4) Hamming code is used to deliver error correction for up to one flipped bit in any block of seven bits, provide an expression for the residual error probability P_e that such a scheme would fail. [2 marks]

(*iii*) If instead repetition were used to try to achieve error correction by repeating every message an odd number of times $N = 2m + 1$, for a positive integer m followed by majority voting, provide an expression for the residual error probability P_e that the repetition scheme would fail. [2 marks]

(*c*) Gabor wavelets are an important class of complex-valued functions for encoding information with maximal resolution simultaneously in the frequency domain and the signal domain. Using an expression for their functional form, explain:





(*i*) their spiral helical trajectory as phasors, shown here with projections of their real and imaginary parts;
(*ii*) the Uncertainty Principle under which they are optimal;
(*iii*) the spaces they occupy in the Information Diagram;
(*iv*) some of their uses in pattern encoding and recognition.

[8 marks]