8 Algorithms (FMS)

(*a*) Explain the greedy strategy in algorithm design. To what problems does it apply? [3 marks]

(*b*) If a problem can be solved with both dynamic programming and a greedy algorithm, what are the advantages of using one or the other? [2 marks]

(*c*) An imaginary post office machine must issue decorative stamps adding up to a given amount of p pence. Its goal is to minimize the number of postage stamps issued, and the machine always has as many stamps as needed.

   (*i*) Let the set of available denominations for the stamps be $D = \{$1p, 5p, 25p, 50p, £1, £2$\}$. Can this problem be solved using bottom-up dynamic programming? If so, clearly describe your algorithm and determine its complexity. If not, prove that it cannot be done. [5 marks]

   (*ii*) Let $c_1 < c_2 < \cdots < c_n$ be n stamp denominations. Prove that if each c_i (a positive integer) is a multiple of c_{i-1} for every $i = 2, \ldots, n$ then the greedy strategy applied to the set $D = \{c_1, c_2, \cdots, c_n\}$ finds the optimal solution for any amount p that is a multiple of c_1. [7 marks]

   (*iii*) Provide a set of denominations for stamps D and an amount of pence p for which the greedy strategy fails to give an optimal solution, p being a multiple of the smallest denomination in D. Show what solution the greedy strategy would find and what the optimal solution is. [3 marks]