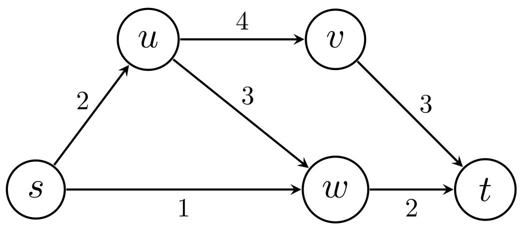
## 10 Algorithms (TMS)

(*a*) State the Max-Flow Min-Cut Theorem. [2 marks]

(*b*) For an arbitrary integer $k \geq 1$, give an example of a flow network with at most five vertices on which the basic Ford-Fulkerson method takes at least $k$ steps to terminate. [4 marks]

(*c*) Consider the following flow network $G$:




Given an initial flow $f$ with $f(s,u) = f(u,w) = f(w,t) = 2$, perform one iteration of Ford-Fulkerson; that is, draw the residual graph $G_f$, specify an augmenting path in $G_f$, and update the flow $f$. Is this new flow a maximum flow? Justify your answer. [5 marks]

(*d*) Given an undirected, connected graph $G = (V, E)$, the edge-connectivity of $G$ is the *size* of a smallest set of edges $X \subseteq E$ so that the graph $G' = (V, E \setminus X)$ becomes disconnected.

(*i*) Describe an algorithm that computes the edge-connectivity of $G$, and analyse its runtime and correctness. [7 marks]

(*ii*) Extend your algorithm so that it also returns a set $X$ satisfying the conditions above. [2 marks]