COMPUTER SCIENCE TRIPOS Part IA – 2014 – Paper 1

9 Algorithms (TMS)

(*a*) Explain the terms *amortized analysis*, *aggregate analysis* and *potential method*. [6 marks]

(*b*) Consider an arbitrary sequence of n stack operations `PUSH()`, `POP()` and `MULTIPOP(x)` in which `POP()` or `MULTIPOP(x)` never attempt to remove more elements than there are on the stack. Assuming that the stack begins with s_0 items and finishes with s_n items, determine the worst-case total cost for executing the n operations as a function of n, s_0 and s_n. You may assume `PUSH()` and `POP()` cost 1 each and `MULTIPOP(x)` costs x. [5 marks]

(*c*) Suppose we want to store a number of items in an array, but we do not know in advance how many items need to be stored. The `INSERT(x)` operation appends an item x to the array. More precisely, if the size of the array is large enough, x is inserted directly at the end of the array. Otherwise, a new array of larger size is created that contains all previous items with x being appended at the end. The total cost of `INSERT(x)` is 1 in the first case, and the size of the new array in the second case.

(*i*) Devise a strategy which, for any integer n, performs any sequence of n `INSERT(.)` operations at a total cost of $O(n)$. [5 marks]

(*ii*) For the strategy described in (*c*)(*i*), give a proof of the cost of the algorithm using the potential method. [4 marks]