9 Semantics of Programming Languages (SS)

This question is about a language that is like L1 but with a stack instead of a store.

(*a*) Consider the following grammars for expressions e and values v:

$$e ::= \mathsf{push}(e) \mid \mathsf{pop}() \mid \mathsf{skip} \mid e_1 \,;\, e_2 \mid \mathsf{true} \mid \mathsf{false} \mid \mathsf{if}\, e_1 \,\mathsf{then}\, e_2 \,\mathsf{else}\, e_3$$
$$v ::= \mathsf{skip} \mid \mathsf{true} \mid \mathsf{false}.$$

The configurations for this language are pairs $\langle e \,,\, bs\rangle$ where e is an expression and bs is a finite list of booleans.

The operational semantics of $\mathsf{push}(e)$ and $\mathsf{pop}()$ are defined by the following rules:

$$\frac{-}{\langle \mathsf{push}(\mathsf{true}) \,,\, bs\rangle \longrightarrow \langle \mathsf{skip} \,,\, (\mathsf{true} :: bs)\rangle} \qquad \frac{\langle e \,,\, bs\rangle \longrightarrow \langle e' \,,\, bs'\rangle}{\langle \mathsf{push}(e) \,,\, bs\rangle \longrightarrow \langle \mathsf{push}(e') \,,\, bs'\rangle}$$

$$\frac{-}{\langle \mathsf{push}(\mathsf{false}) \,,\, bs\rangle \longrightarrow \langle \mathsf{skip} \,,\, (\mathsf{false} :: bs)\rangle} \qquad \frac{-}{\langle \mathsf{pop}() \,,\, b :: bs\rangle \longrightarrow \langle b \,,\, bs\rangle}$$

Write down rules for the other language constructs, to define a reasonable operational semantics. [5 marks]

(*b*) The types for this language are

$$T ::= \mathsf{unit} \mid \mathsf{bool}$$

We define a relation $e : T$ between expressions and types. The types of $\mathsf{push}(e)$ and $\mathsf{pop}()$ are given by the following rules:

$$\frac{e : \mathsf{bool}}{\mathsf{push}(e) : \mathsf{unit}} \qquad \frac{-}{\mathsf{pop}() : \mathsf{bool}}$$

Write down rules for the other language constructs to define a reasonable type system. [5 marks]

(*c*) Consider the following statements:

(*i*) For all pairs of configurations $\langle e \,,\, bs\rangle$, $\langle e' \,,\, bs'\rangle$, and all types T: if $e : T$ and $\langle e \,,\, bs\rangle \longrightarrow \langle e' \,,\, bs'\rangle$ then $e' : T$.

(*ii*) For all configurations $\langle e \,,\, bs\rangle$ and all types T: if $e : T$ then either e is a value or there is a configuration $\langle e' \,,\, bs'\rangle$ such that $\langle e \,,\, bs\rangle \longrightarrow \langle e' \,,\, bs'\rangle$.

For each of these two statements, state whether it holds. If it holds, prove it. If it doesn't hold, explain why and suggest a change to the semantics that would make the theorem hold. [10 marks]