COMPUTER SCIENCE TRIPOS Part IB – 2013 – Paper 6

1 Complexity Theory (AD)

Consider the following decision problems:

> **Prime**: given a positive integer n written in binary, determine whether n is prime.

> **Factor**: given positive integers n and k, written in binary, determine whether n has a factor h such that $1 < h < k$.

> **Subset Sum**: given a collection of positive integers $a_1, \ldots, a_l$ and t determine whether there is a set $S \subseteq \{1, \ldots, l\}$ such that $\sum_{i \in S} a_i = t$.

(a) For each of the three problems above and each of the complexity classes P, NP and co-NP, state what complexity classes the problem is in, with brief justification. [8 marks]

(b) For each of the following statements, state whether it is true, false, or unknown. In each case, give justification for your answer and in the case where the truth of the statement is unknown, state any implications that might follow from it being true or false.

 (i) **Prime** is NP-complete.

 (ii) **Factor** is NP-complete.

 (iii) Since multiplication and factorization are inverses of each other, multiplication is a one-way function.

 (iv) **Factor** is in PSpace.

[3 marks each]