5   Logic and Proof (LCP)

(*a*) Exhibit a model for the following set of formulas, or prove that none exists.

$$P \to Q \wedge R \qquad P \wedge Q \to S \qquad \neg R \leftrightarrow S \qquad P \vee Q$$

[8 marks]

(*b*) Consider the following set of clauses:

$$\{\neg(x < y),\ -y < -x\} \qquad \{\neg(x < y),\ x + z < y + z\} \qquad \{0 < 1\}$$

(*i*)   What is the Herbrand universe of these clauses?   [3 marks]

(*ii*) What semantics must any Herbrand interpretation of the clauses attach to the function symbols?   [3 marks]

(*iii*) Specify an Herbrand model by giving a semantics of the relation $<$, justifying your choice with reference to a natural model of the set of clauses.
[6 marks]