7 Probability (RJG)

(*a*) A biased coin has probability p, $0 < p < 1$, of showing heads on a single throw. Show that the probability generating function of the random variable, X, giving the number of heads in n independent throws, is given by

$$G_X(z) = (pz + 1 - p)^n$$

[4 marks]

(*b*) Now suppose that the coin is thrown N times where N is a random variable with $\mathbb{E}(N) = \mu_N$ and $\mathrm{Var}(N) = \sigma_N^2$ and let Y be the random number of heads obtained.

(*i*) Show that

$$G_Y(z) = G_N(pz + 1 - p)$$

where $G_N(z)$ is the probability generating function of N. [4 marks]

(*ii*) Find $\mathbb{E}(Y)$ and $\mathrm{Var}(Y)$. [4 marks]

(*c*) Suppose that N has a Poisson distribution with parameter $\lambda > 0$.

(*i*) Find $G_N(z)$. [4 marks]

(*ii*) Show that Y has a Poisson distribution with parameter λp. [4 marks]