4 Discrete Mathematics I (SS)

(*a*) Let A be the set $\{1,2,3\}$. The following relations are subsets of $A \times A$. Draw them as directed graphs.

(*i*) $R_1 = \{(x,y) \mid x \in A \wedge y \in A \wedge x - y = 1\}$

(*ii*) $R_2 = \{(x,y) \mid x \in A \wedge y \in A \wedge x - y \geq 1\}$

(*iii*) $R_3 = \{(x,y) \mid x \in A \wedge y \in A \wedge x - y = 0\}$

(*iv*) $R_4 = \{(x,y) \mid x \in A \wedge y \in A \wedge \neg(x - y = 0)\}$

(*v*) $R_5 = \{(x,y) \mid x \in A \wedge y \in A \wedge \forall u.\, \exists v.\, x + u = y + v\}$
where u and v range over the integers

(*vi*) $R_6 = \{(x,y) \mid x \in A \wedge y \in A \wedge \exists u.\, \forall v.\, x + u = y + v\}$
where u and v range over the integers [6 marks]

(*b*) Write down what it means for a relation to be transitive. Which of the relations in part (*a*) are transitive? [3 marks]

(*c*) Write down the introduction and elimination rules for the universal quantifier in structured proof. [3 marks]

(*d*) Recall the following introduction and elimination rules for implication.

. . .

m. Assume P
. . .
n. Q from . . . by . . .

$n+1$. $P \Rightarrow Q$ from m–n,
by $\Rightarrow$-introduction.

. . .
l. $P \Rightarrow Q$ from . . . by . . .
. . .
m. P from . . . by . . .
. . .
n. Q from l, m
by $\Rightarrow$-elimination.

Write down a structured proof of the following statement.

$$\big(\forall a.\, P(a) \Rightarrow Q(a)\big) \Rightarrow \Big(\big(\forall b.\, Q(b) \Rightarrow R(b)\big) \Rightarrow \big(\forall c.\, P(c) \Rightarrow R(c)\big)\Big)$$

[8 marks]