2011 Paper 8 Question 2

Hoare Logic

The programming language **L** consists of commands C composed from assignments $V{:=}E$ (where E is an expression) using sequences $C_1;C_2$, conditionals `IF` S `THEN` C_1 `ELSE` C_2 (where S is statement) and while-loops `WHILE` S `DO` C.

(a) Devise a command `SKIP` in **L** that has no effect and, for arbitrary P, prove using the Hoare logic axioms and rules for the constructs of **L** that $\vdash \{P\}\texttt{SKIP}\{P\}$. [4 marks]

(b) Devise a one-armed conditional `IF` S `THEN` C built only from S, C and constructs of **L** and show using the Hoare logic for **L** that if $\vdash \{P \wedge S\}C\{Q\}$ and $\vdash P \wedge \neg S \Rightarrow Q$ then $\vdash \{P\}\texttt{IF}\ S\ \texttt{THEN}\ C\{Q\}$. [6 marks]

(c) Define a command `MAGIC` in **L** that has the property $\vdash \{P\}\texttt{MAGIC}\{Q\}$ for any precondition P and postcondition Q. Prove that your definition of `MAGIC` has this property using the Hoare logic for **L**. [10 marks]