2011 Paper 2 Question 1

Digital Electronics

(a) Simplify the following expressions using Boolean algebra:

(i) $F = A \,.\, \overline{B} \,.\, \overline{C} + \overline{A} \,.\, B \,.\, \overline{C} + \overline{A} \,.\, \overline{B} \,.\, C + A \,.\, B \,.\, C$

(ii) $F = (X + Y) \,.\, (\overline{X} + Y + Z) \,.\, (\overline{X} + Y + \overline{Z})$

(iii) $F = (A \,.\, D + \overline{A} \,.\, C) \,.\, [\overline{B} \,.\, (C + B \,.\, \overline{D})]$

[6 marks]

(b) Give the truth table for an encoder that accepts a sign bit, S, and two magnitude bits X_0, X_1 and gives a three-bit output Y_2, Y_1, Y_0 that are the two's complement encoding of the input. [4 marks]

(c) Using a Karnaugh map, simplify the following expression to yield a solution in a sum-of-products form:

$$Y = \overline{A} \,.\, \overline{B} \,.\, \overline{C} \,.\, D + A \,.\, \overline{B} \,.\, \overline{C} \,.\, D + A \,.\, \overline{D} + \overline{A} \,.\, B \,.\, \overline{D}$$

What problem may exist with a practical realisation of this solution, and how may it be cured? [5 marks]

(d) Simplify the following expression using a Karnaugh map to yield a solution in product-of-sums form and implement it using only NOR gates assuming complemented input variables are available:

$$Y = (B + \overline{C} + \overline{D}) \,.\, (\overline{A} + B + \overline{C}) \,.\, (A + B + \overline{D}) \,.\, (A + \overline{B} + \overline{C})$$

Neglect any potential problems in the practical realisation of your solution. [5 marks]