

2010 Paper 9 Question 5

Denotational Semantics

Indicate whether the following statements are true or false. Provide an argument for each answer. You may use standard results provided that you state them clearly.

(a) For all PCF-types τ and closed PCF-terms M_1 and M_2 of type τ ,

if $\llbracket M_1 \rrbracket = \llbracket M_2 \rrbracket$ in $\llbracket \tau \rrbracket$ then $M_1 \cong_{\text{ctx}} M_2 : \tau$ [6 marks]

(b) For all PCF-types τ and closed PCF-terms M_1 and M_2 of type τ ,

if $M_1 \cong_{\text{ctx}} M_2 : \tau$ then $\llbracket M_1 \rrbracket = \llbracket M_2 \rrbracket$ in $\llbracket \tau \rrbracket$ [7 marks]

(c) For all closed PCF-terms M_1 and M_2 of type $\text{nat} \rightarrow \text{nat}$,

if $M_1 \cong_{\text{ctx}} M_2 : \text{nat} \rightarrow \text{nat}$ then $\llbracket M_1 \rrbracket = \llbracket M_2 \rrbracket$ in $(\mathbb{N}_\perp \rightarrow \mathbb{N}_\perp)$ [7 marks]