2010 Paper 7 Question 8

Denotational Semantics

(*a*) For a domain D, recall that by Tarski's Fixed-Point Theorem every continuous function $f \in (D \to D)$ has a least pre-fixed point $\mathit{fix}(f) \in D$.

Prove that the function $\mathit{fix} : (D \to D) \to D$ is continuous. [10 marks]

(*b*) For a partially ordered set $(P, \sqsubseteq)$, let $\big(\mathrm{Ch}(P), \sqsubseteq_{\mathrm{ptw}}\big)$ be the partially ordered set of chains in P ordered pointwise. That is,

$$\mathrm{Ch}(P) \stackrel{\mathrm{def}}{=} \left\{ x = \{x_n\}_{n \in \mathbb{N}} \;\middle|\; \text{for all } i \leq j \text{ in } \mathbb{N},\ x_i \sqsubseteq x_j \text{ in } P \right\}$$

and

$$x \sqsubseteq_{\mathrm{ptw}} x' \stackrel{\mathrm{def}}{\iff} x_n \sqsubseteq x'_n \text{ for all } n \in \mathbb{N}$$

Show that if P is a domain then so is $\mathrm{Ch}(P)$. [10 marks]