COMPUTER SCIENCE TRIPOS Part IA

Tuesday 1 June 2010 1.30 to 4.30

COMPUTER SCIENCE Paper 2

*Answer **one** question from each of Sections A, B and C, and **two** questions from Section D.*

*Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags

SPECIAL REQUIREMENTS
Approved calculator permitted

SECTION A

1 Digital Electronics

(a) In a particular computer system, numbers are represented using words having a length of 4 bits.

(i) What is the range of positive numbers that can be represented using unsigned binary numbers? [2 marks]

(ii) Explain how the 2's complement representation can be used to describe signed binary numbers, using 4-bit words as an example. [3 marks]

(iii) Using decimal (base 10) representation for the answers, perform the following 2's complement 4-bit additions, noting any problems:

- $0110 + 1101$
- $1010 + 1011$ [4 marks]

(b) Complete the following truth table that describes a single-bit full adder:

C_{IN}	A	B	C_{OUT}	sum
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	
1	0	0	0	
1	0	1	1	
1	1	0	1	
1	1	1	1	

where C_{IN} is carry-in, A and B are the input data, C_{OUT} is carry-out and sum is the sum output. Remember to write your answer on the script paper, i.e. not on the question paper. [2 marks]

(c) Show how 4 single-bit full-adders can be combined to implement a 4-bit ripple carry-adder. [2 marks]

(d) Briefly describe how the speed of operation of the approach in part (c) could be improved. [4 marks]

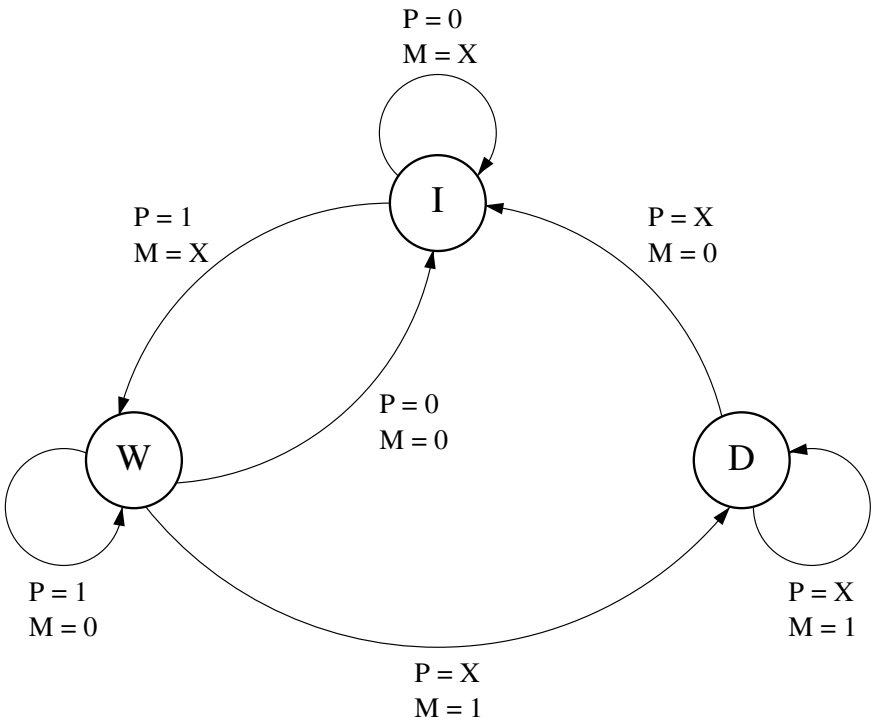
(e) Show how C_{OUT} in part (b) can be implemented using only NAND gates. [3 marks]

2 Digital Electronics

(*a*) With the aid of appropriate diagrams, briefly explain the operation of Moore and Mealy finite state machines and highlight their differences. [7 marks]

(*b*) A Boolean output Y is to be set to binary "1" for one clock cycle when the sequence 10010 is detected in a stream of serial digital data. The system is only to respond to non-overlapping sequences. Draw the state diagram for this system. [6 marks]

(*c*) A machine is in one of three states: Idle (I), Working (W) and Down (D). The machine has two binary inputs: "processing" (P) and "malfunction" (M). The state diagram of the machine is as follows, where X means "don't care"



and the state assignment I = [00], W = [01] and D = [10]. Note that state = $[Q_1Q_2]$ where Q_n is the output flop flip-flop n.

(*i*) Write down the state table. [3 marks]

(*ii*) Assuming the use of D-type flip-flops for the state registers, derive the minimised Boolean expressions for the next-state functions. [4 marks]

SECTION B

3 Operating Systems

(*a*) Shortest Job First (SJF), Shortest Remaining Time First (SRTF) and Round Robin (RR) are CPU scheduling algorithms. Briefly describe each. [1 mark each]

(*b*) Consider the following set of processes.

Process	Creation Time	Required Computing Time
P_1	0	25
P_2	5	15
P_3	10	5
P_4	15	5

(*i*) Draw a diagram showing how SJF would schedule the processes. What is the average waiting time? [3 marks]

(*ii*) Draw a diagram showing how SRTF would schedule the processes. What is the average waiting time in this case? [3 marks]

(*iii*) Assuming a quantum of 10 time units, draw a diagram showing how RR would schedule the processes. What is the average waiting time in this case? [3 marks]

(*iv*) A student suggests that RR could be improved by reducing the quantum to 1 time unit. What are the advantages and disadvantages of this proposal? [2 marks]

(*c*) Compare and contrast the Unix *buffer cache* and the Windows NT *cache manager*. [6 marks]

4 Operating Systems

(*a*) The virtual address space of a UNIX V7 process contains a *text segment*, a *data segment* and a *stack segment*.

(*i*) What is contained in the text segment? How does this change as the process executes? [2 marks]

(*ii*) What is contained in the data segment? How does this change as the process executes? [2 marks]

(*iii*) What is contained in the stack segment? How does this change as the process executes? [2 marks]

(*b*) The UNIX kernel is also present in the virtual address space of every process. Describe how the operating system can ensure that this memory region is protected from access by an executing process. Under what circumstances can a process gain access to this region of virtual memory? [2 marks]

(*c*) Compare and contrast *blocking*, *non-blocking* and *asynchronous* I/O. [2 marks each]

(*d*) You are asked to write a device driver for a hard-disk drive.

(*i*) Under what circumstances will you issue requests to the drive? [2 marks]

(*ii*) What steps will you need to take when an interrupt occurs? [2 marks]

(*iii*) Given that the hard-disk drive is not really a random access device, what steps could you take to improve performance? [2 marks]

SECTION C

5 Discrete Mathematics II

The set Ω is characterised as the least set such that

$$\text{if } X \text{ is finite and } X \subseteq \Omega\,, \text{ then } X \cup \bigcup X \in \Omega$$

[Recall $\bigcup X =_{\text{def}} \{y \mid \exists x \in X.\, y \in x\}$.]

(a) Show

(i) $\emptyset \in \Omega$, and [2 marks]

(ii) if $x \in \Omega$ then $\{x\} \cup x \in \Omega$. [3 marks]

(b) State the rule(s) and the principle of rule induction appropriate for the set Ω. [3 marks]

(c) Define a set x to be *transitive* iff

$$\forall z, y.\ z \in y\ \&\ y \in x \Rightarrow z \in x$$

Prove that all elements of Ω are transitive. [8 marks]

(d) Describe, without proof, the elements of the set Ω. [4 marks]

6 Discrete Mathematics II

A preorder $(E, \lesssim)$ consists of a set E on which there is a binary relation $\lesssim$ which is reflexive and transitive.

(a) A subset $x \subseteq E$ is *down-closed* iff

$$\forall e, e' \in E.\ e \in x\ \&\ e' \lesssim e\ \Rightarrow\ e' \in x$$

Write $\mathcal{D}(E)$ for the set of all down-closed subsets of E.

Show that

(i) $\emptyset \in \mathcal{D}(E)$, and [1 mark]

(ii) if $X \subseteq \mathcal{D}(E)$ then $\bigcup X \in \mathcal{D}(E)$. [3 marks]

[Recall $\bigcup X =_{\text{def}} \{e \in E \mid \exists x \in X.\, e \in x\}$.]

(b) An element $p \in \mathcal{D}(E)$ is said to be a *complete prime* iff

$$\forall X \subseteq \mathcal{D}(E).\ p \subseteq \bigcup X\ \Rightarrow\ \exists x \in X.\ p \subseteq x$$

For $e \in E$, define $[e]$ to be the down-closed set $\{e' \in E \mid e' \lesssim e\}$.

(i) Is $\emptyset$ a complete prime? Justify your answer. [3 marks]

(ii) Show for all $x \in \mathcal{D}(E)$ that

$$x = \bigcup \{[e] \mid e \in x\}$$

[3 marks]

(c) For $x \in \mathcal{D}(E)$, show that x is a complete prime iff $x = [e]$ for some $e \in E$. [Remember to show both directions of the "iff".] [6 marks]

(d) For $x \in \mathcal{D}(E)$, define

$$N(x) = \bigcup \{w \in \mathcal{D}(E) \mid w \cap x = \emptyset\}$$

Show, for $e \in E$, that

$$e \in N(x)\ \text{ iff }\ \forall e' \lesssim e.\ e' \notin x$$

[4 marks]

SECTION D

7 Professional Practice and Ethics

(*a*) Consider **two** ethical principles of your choice. What is the best justification of these principles? Consider some reasonable alternative justification of these principles. Why is your justification better? (Remember, if you think your justification is the best, then you must have some reason for thinking so!) [6 marks]

(*b*) Give **four** characteristics that distinguish a professional body from other social organisations. [4 marks]

(*c*) The Computer Misuse Act was passed in 1990. What shortcomings of the law at the time led to this new Act? How was this Act amended by the Police and Justice Bill of 2006, and why were these amendments introduced? Which aspect of the Police and Justice Bill raises problems for Security professionals who test for security vulnerabilities in computing systems, and why? [4 marks]

(*d*) The Seventh and Eighth Principles of the Data Protection Act concern data security. The Seventh concerns security within the borders of the UK, the Eighth concerns security outside the borders of the UK. State **two** of the requirements of the Seventh Principle and **one** of the Eighth Principle. [3 marks]

(*e*) Offer the best argument you can **either** for the institution of proprietary software, **or** for the institution of free (as in free speech) software. [3 marks]

8 Probability

(a) A coin that comes up "heads" with probability p is tossed n times independently.

(i) What is the likelihood that k of these n tosses will be "heads", and the remainder "tails"? [2 marks]

(ii) Give the mean and the variance expected for the number of "heads" outcomes. [1 mark each]

(b) In a different experiment with this same coin, you monitor how many tosses are needed before getting the *first* outcome of a "head".

(i) What is the likelihood that the *first* "head" occurs on the k^{th} trial? [2 marks]

(ii) What is the mean trial number k for the first "head", and what is the variance for this number? [1 mark each]

(c) In a Poisson process with hazard parameter λ:

(i) What is the likelihood of observing k events? [2 marks]

(ii) What is the mean, and what is the variance, expected for the number of observed events? [1 mark each]

(d) If X and Y are random variables having expectations $E(X)$ and $E(Y)$ respectively:

(i) What is their covariance $Cov(X, Y)$? [2 marks]

(ii) In terms of their covariance $Cov(X, Y)$, and their respective variances $Var(X)$ and $Var(Y)$, what is their correlation coefficient $\rho(X, Y)$? [2 marks]

(e) For a continuous random variable X that is exponentially distributed, having density function $f(x) = \lambda \exp(-\lambda x)$ for $x > 0$ and $f(x) = 0$ for $x \leq 0$:

(i) Derive the expectation $E(X)$ of this random variable. [2 marks]

(ii) Derive its variance $Var(X)$. [2 marks]

9 Regular Languages and Finite Automata

(a) Let M be a finite automaton and let M' be obtained from M by interchanging the collections of accepting and non-accepting states.

(i) What does it mean for M to be *deterministic*? [2 marks]

(ii) If M is deterministic, explain why the language accepted by M' is the complement of the language accepted by M. [3 marks]

(iii) Give an example, with justification, to show that the property in part (ii) can fail to hold if M is non-deterministic. [2 marks]

(b) Draw pictures of non-deterministic finite automata with ε-transitions over input alphabet $\{a, b\}$ whose languages of accepted strings are

(i) $\{a, aa, aaa\}$ [1 mark]

(ii) all strings not in $\{a, aa, aaa\}$ [3 marks]

(iii) all strings whose length is divisible by 3 or 5 [3 marks]

(iv) all strings matching the regular expression $(aa|b)^*(bb|a)^*$ [3 marks]

(v) all strings not matching the regular expression $(\emptyset^*)^*$ [3 marks]

END OF PAPER