2009 Paper 1 Question 3

Discrete Mathematics I

(a) State the structured-proof rules for implication introduction and disjunction elimination. [3 marks]

(b) Give either a structured proof or a counterexample for each of the following.

(i) $((P \Rightarrow Q) \vee (P \Rightarrow R)) \Rightarrow (P \Rightarrow (Q \vee R))$

(ii) $((P \wedge Q) \Rightarrow R) \Rightarrow ((P \Rightarrow R) \wedge (Q \Rightarrow R))$

[8 marks]

For a set of sets A, write $\bigcup A$ for the set $\{x \mid \exists X \in A . x \in X\}$. For a non-empty set of sets A, write $\bigcap A$ for the set $\{x \mid \forall X \in A . x \in X\}$.

(c) Suppose $A \subseteq \mathcal{P}(X)$ and $B \subseteq \mathcal{P}(X)$. Prove or give a counterexample for each of the following.

(i) If $\bigcup A$ and $\bigcup B$ are disjoint, then A and B are disjoint.

(ii) If A and B are disjoint then $\bigcup A$ and $\bigcup B$ are disjoint.

(iii) If A and B are non-empty and $\forall X \in A . \forall Y \in B . X \subseteq Y$ then $\bigcup A \subseteq \bigcap B$.

[9 marks]