COMPUTER SCIENCE TRIPOS Part IA

Tuesday 2 June 2009 1.30 to 4.30

COMPUTER SCIENCE Paper 2

*Answer **one** question from each of Sections A, B and C, and **two** questions from Section D.*

*Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags

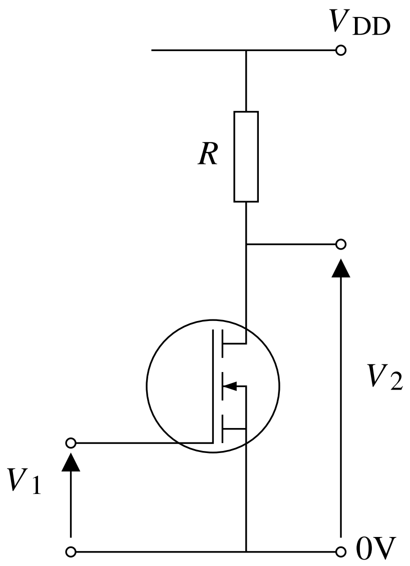
SPECIAL REQUIREMENTS
Approved calculator permitted

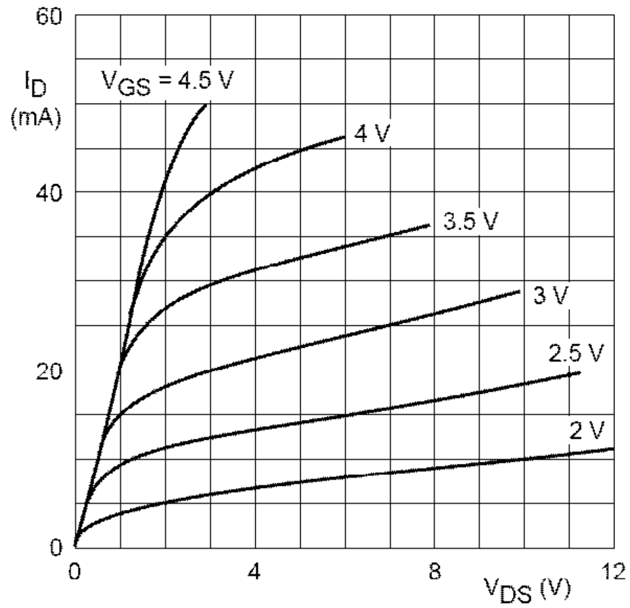
SECTION A

1 Digital Electronics

(*a*) With the aid of appropriate sketches, describe how an n-channel MOSFET operates as a switch. [6 marks]

(*b*) The left-hand figure below shows a circuit that uses an n-channel MOSFET having the properties given in the right-hand figure. The supply voltage $V_{DD} = 10V$ and the resistor $R = 200\Omega$. The circuit input and output voltages are V_1 and V_2 respectively.





(*i*) Find the corresponding values of V_2 when $V_1 = 0V$, 2V, 2.5V, 3V, 3.5V, 4V, 4.5V, and sketch V_2 as a function of V_1. What logical function does this circuit implement? [8 marks]

(*ii*) When $V_1 = 4.5V$, calculate the power dissipated by the entire circuit and by resistor R. [2 marks]

(*c*) With the aid of a circuit diagram, describe how a p-channel MOSFET can be used in a modified version of the left-hand figure above to significantly reduce total power dissipation. [4 marks]

2 Digital Electronics

(*a*) With the aid of a suitable diagram, explain *set-up time*, *hold time* and *propagation delay* for a positive edge triggered D-type flip-flop. [6 marks]

(*b*) The controller of a car wash machine is designed to produce the following sequence of steps.

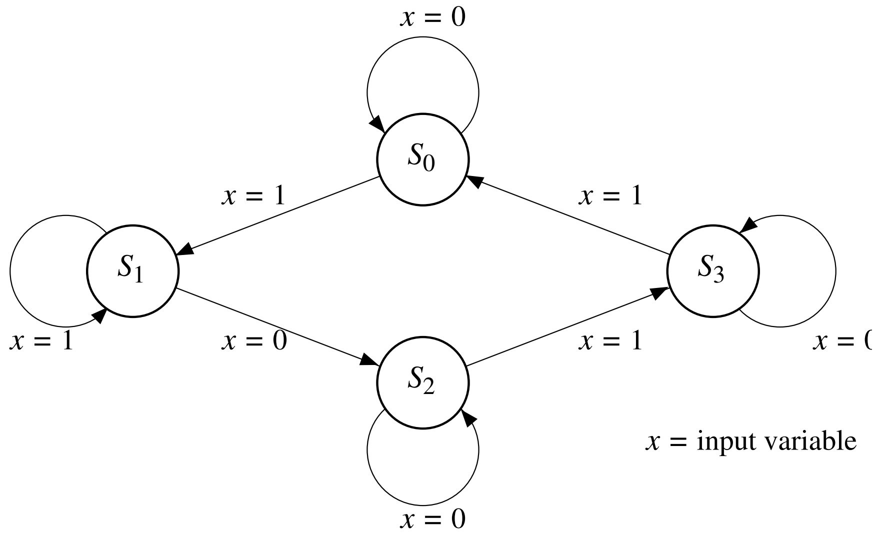
Water spray (W)	Sponge (S)	Heater (H)
0	0	0
1	0	0
1	1	0
0	0	1
0	0	0

The sequence starts at $W = S = H = 0$ following the pressing of a button B: i.e. $B = 1$ if pressed, $B = 0$ otherwise.

If B is pressed while the heater is on ($H = 1$) then return to the step with the heater off ($H = 0$) and water spray on ($W = 1$) and sponge on ($S = 1$). Otherwise B has no effect until the entire sequence of steps is complete.

Draw a state diagram for the system. [6 marks]

(*c*) Consider the following state diagram



and the state assignment $S_0 = 00$, $S_1 = 01$, $S_2 = 10$ and $S_3 = 11$. Write down the state table. Assuming the use of D-type flip-flops for the state registers, derive the minimised Boolean expressions for the next-state functions. Note that state $= (Q_1, Q_0)$ where Q_n is the output from flip-flop n. [8 marks]

SECTION B

3 Operating Systems

(*a*) Operating systems typically provide each process with a *virtual address space*.

(*i*) Give *three* advantages of this. [3 marks]

(*ii*) In which circumstances does *external fragmentation* occur? How can it be managed? [2 marks]

(*iii*) In which circumstances does *internal fragmentation* occur? [1 mark]

(*iv*) Design a multi-level page table for a computer with a 48-bit virtual address space, 48-bit physical address space, and a 4K page size. You should explain its operation, and justify your design decisions. [6 marks]

(*b*) In the context of the UNIX operating system:

(*i*) What is a *pipe*? What is it used for? How does it operate? Use a diagram to illustrate your answer. [4 marks]

(*ii*) What is the *shell*? Describe its operation in pseudo-code, giving special emphasis to any system calls invoked. [4 marks]

4 Operating Systems

(*a*) In the context of virtual memory management:

(*i*) What is *demand paging*? How is it implemented? [4 marks]

(*ii*) What is meant by *temporal locality of reference*? [2 marks]

(*iii*) How does the assumption of temporal locality of reference influence page replacement decisions? Illustrate your answer by briefly describing an appropriate page replacement algorithm or algorithms. [3 marks]

(*iv*) What is meant by *spatial locality of reference*? [2 marks]

(*v*) In what ways does the assumption of spatial locality of reference influence the design of the virtual memory system? [3 marks]

(*b*) Buses are used to connect devices to the processor.

(*i*) Describe with the aid of a diagram the operation of a *synchronous* bus. [4 marks]

(*ii*) In what ways does an *asynchronous* bus differ? [2 marks]

SECTION C

5 Discrete Mathematics II

The set S of strings over symbols a and b is defined to be the least set S of strings such that

$$a \in S \ ,$$

$$as \in S \ \text{ if } s \in S \ , \ \text{ and}$$

$$bst \in S \ \text{ if } s \in S \text{ and } t \in S \ .$$

(*a*) The set S may also be described as the least subset of strings closed under certain rules. Describe the rules. Write down a principle of rule induction appropriate for the set S. [5 marks]

(*b*) Exhibit a derivation, indicating which rules are used, to show that the string *aabbaaa* is in S. [4 marks]

(*c*) For a string s, let $N_a(s)$ denote the number of occurrences of a in s, and similarly, let $N_b(s)$ denote the number of occurrences of b. Prove for every string $s \in S$ that $N_a(s) > N_b(s)$, i.e. there are strictly more occurrences of a than occurrences of b. [5 marks]

(*d*) Exhibit a string that has strictly more occurrences of a than occurrences of b and yet is not in S. Prove that your example string is not in S. [6 marks]

6 Discrete Mathematics II

(*a*) A partial order $(P, \leq)$ comprises a set P together with a binary relation $\leq$ which is reflexive, transitive and antisymmetric. Explain what the terms *reflexive*, *transitive* and *antisymmetric* mean. [3 marks]

(*b*) The relation $\leq$ on natural numbers $\mathbb{N} = \{1, 2, \cdots\}$ is defined by

$$m \leq n \text{ iff } m \text{ divides } n, \text{ that is } m \cdot k = n \text{ for some integer } k\,.$$

Invoking standard facts about division, establish that $\leq$ is a partial order. If in the definition of $\leq$ we used the set of all integers $\mathbb{Z}$, instead of $\mathbb{N}$, would $(\mathbb{Z}, \leq)$ be a partial order? Explain your answer briefly. [5 marks]

(*c*) Draw the Hasse diagram for $\leq$ on the set $\{1, 2, \cdots, 13\}$. Identify the greatest lower bound (glb) and least upper bound (lub) of $\{4, 6\}$. Does the partial order $(\mathbb{N}, \leq)$ have greatest lower bounds and least upper bounds of all subsets of $\mathbb{N}$, including all infinite subsets? Explain your answers briefly. [6 marks]

(*d*) An *atom* of the partial order $(\mathbb{N}, \leq)$ is an element $a \in \mathbb{N}$ such that

$$\forall x \in \mathbb{N}.\ (1 \leq x \text{ and } x \leq a) \Rightarrow (1 = x \text{ or } x = a)\,.$$

Identify the atoms in your Hasse diagram, and more generally in $\mathbb{N}$. [3 marks]

(*e*) Explain, without proof, why a partial order that has least upper bounds of all subsets also has greatest lower bounds of all subsets. [3 marks]

SECTION D

7 Professional Practice and Ethics

(*a*) State *two* problems with consequentialist theories. [2 marks]

(*b*) What are the *two* main kinds of deontological theory? [2 marks]

(*c*) Name *two* of the several kinds of relationship in which a professional is likely to be involved and indicate one ethical dimension in each of these relations. [4 marks]

(*d*) The last section of the British Computer Society Code of Conduct concerns professional competence and integrity. Indicate *two* kinds of conduct that the Code requires to maintain professional competence and integrity. [2 marks]

(*e*) How has the concept of computer hacking changed from its use before the advent of the Internet and today? [2 marks]

(*f*) What is *social engineering* in the context of computer cracking, and what are the remedies for it? [2 marks]

(*g*) The definition of privacy is still open to debate. What would you consider an important criterion of privacy (not necessarily the only one), and how would it contribute to privacy? [2 marks]

(*h*) The Data Protection Act of 1998 specifies several kinds of information that must be provided to the data subject. Indicate *two* of these kinds of information. [2 marks]

(*i*) What is the basic dilemma raised by legally protecting intellectual property? [2 marks]

8 Probability

(*a*) Consider a random variable, X, taking non-negative integer values.

(*i*) Define the *probability generating function*, $G_X(z)$, of the random variable X. [2 marks]

(*ii*) Derive the expression for the expectation, $\mathbb{E}(X)$, in terms of the first derivative of $G_X(z)$. [2 marks]

(*b*) Calculate $G_X(z)$ in the following two cases.

(*i*) Suppose that X takes values equally likely from the set $\{0, 1, 2, 3, 4, 5\}$. [2 marks]

(*ii*) Suppose that X has the Binomial distribution $Bin(n, p)$ where $0 \leq p \leq 1$ and n a positive integer. [2 marks]

(*c*) Suppose that X and Y are two independent random variables each taking non-negative integer values and let their probability generating functions be $G_X(z)$ and $G_Y(z)$, respectively. Show that $X + Y$ has a probability generating function, $G_{X+Y}(z)$, given by

$$G_{X+Y}(z) = G_X(z)G_Y(z)\,.$$

[4 marks]

(*d*) Suppose that X and Y are independent random variables with the marginal distributions $Bin(n_1, p_1)$ and $Bin(n_2, p_2)$, respectively.

(*i*) Find the generating function $G_{X+Y}(z)$ and the expectation, $\mathbb{E}(X + Y)$. [4 marks]

(*ii*) Under what conditions on the parameters n_1, p_1 and n_2, p_2 is $X + Y$ again a Binomial distribution? [4 marks]

9 Regular Languages and Finite Automata

Let L be a language over an alphabet Σ. The equivalence relation $\sim_L$ on the set Σ^* of finite strings over Σ is defined by $u \sim_L v$ if and only if for all $w \in \Sigma^*$ it is the case that $uw \in L$ if and only if $vw \in L$.

(a) Suppose that $L = L(M)$ is the language accepted by a deterministic finite automaton M. For each $u \in \Sigma^*$, let $s(u)$ be the unique state of M reached from the initial state after inputting the string u. Show that $s(u) = s(v)$ implies $u \sim_L v$. Deduce that for this L the number of $\sim_L$-equivalence classes is finite. [Hint: if M has n states, show that no collection of equivalence classes can contain more than n distinct elements.] [10 marks]

(b) Suppose that $\Sigma = \{a, b\}$ and L is the language determined by the regular expression $a^*b(a|b)$. Using part (a), or otherwise, give an upper bound for the number of $\sim_L$-equivalence classes for this L. [5 marks]

(c) Suppose that $\Sigma = \{a, b\}$ and $L = \{a^n b^n \mid n \geq 0\}$. By considering a^n for $n \geq 0$, or otherwise, show that for this L there are infinitely many different $\sim_L$-equivalence classes. [5 marks]

END OF PAPER