2008 Paper 9 Question 14

Denotational Semantics

(*a*) Show that every continuous function $f : D \to D$ on a domain D has a least prefixed point, $\mathit{fix}(f)$. [3 marks]

(*b*) Let $h : P \to P$ be a continuous function on a domain P. Show that $\mathit{fix}(h) = \mathit{fix}(h \circ h)$. [3 marks]

(*c*) Let D be a domain. Let $f : D \to D$ and $g : D \to D$ be continuous functions. Define the continuous function $h : D \times D \to D \times D$ by

$$h(x, y) = (g(y), f(x))$$

for $x, y \in D$. Show

$$\mathit{fix}(h) = (\mathit{fix}(g \circ f), \mathit{fix}(f \circ g))$$

[4 marks]

(*d*) Define what it means for two terms of PCF to be *contextually equivalent*. Exhibit two terms of PCF that are contextually equivalent yet have distinct denotations in the domain $(\mathbb{B}_\perp \to (\mathbb{B}_\perp \to \mathbb{B}_\perp)) \to \mathbb{B}_\perp$ where $\mathbb{B} = \{\mathit{true}, \mathit{false}\}$ is the set of truth values. Exhibit the domain element on which the denotations differ. [10 marks]