2008 Paper 8 Question 11

Digital Signal Processing

(a) What is the Fourier transform of a rectangular pulse of amplitude A and duration $d > 0$, centred around $t = 0$? [4 marks]

(b) Calculate the Fourier transform of the triangular pulse

$$\Lambda(t) = \begin{cases} 1 - |t|, & \text{for } |t| < 1 \\ 0, & \text{otherwise} \end{cases}$$

[Hint: Think of $\Lambda(t)$ as the result of a convolution.] [4 marks]

(c) A 2 kHz sine wave is sampled at 12 kHz. The resulting values are later converted back into a continuous signal using *linear interpolation*.

  (i) At what other frequencies besides 2 kHz is there signal energy in the resulting continuous waveform? [4 marks]

  (ii) Consider among those other components the one with the lowest frequency. By what factor is its voltage lower compared with the 2 kHz component? [4 marks]

  (iii) Your colleague records with a PC soundcard at 44.1 kHz sampling frequency 1024 samples of the continuous waveform, loads these into MATLAB as vector x and then attempts to plot an amplitude spectrum with the command

  plot(real(fft(x)));

  Name *two* problems that need to be fixed in this command before the resulting plot is likely to agree with the result of (*ii*). [4 marks]