

2008 Paper 6 Question 9

Foundations of Functional Programming

(a) For λ -terms, define

(i) α -equivalence;

(ii) β -reduction;

(iii) η -reduction;

(iv) equality.

You may assume appropriate syntactic definitions such as FV (free variables), variable swapping, and capture-avoiding substitution. [8 marks]

(b) Prove or disprove the following equalities:

(i) $(\lambda x. (\lambda y. x y)) (\lambda x. y) = \lambda z. y$ [3 marks]

(ii) $(\lambda x. M) ((\lambda y. N) P) = (\lambda y. (\lambda x. M) N) P$ where $y \notin FV(M)$.

[Hint: If $y \notin FV(M)$, then $\forall L. M[L/y] \equiv M$.] [4 marks]

(c) Show how adding the following reduction rule makes all λ -terms equal:

$$M N \rightarrow_s N M$$

[Hint: You may find it useful to recall $(\lambda x. M) L = M$ if $x \notin FV(M)$.] [5 marks]