## 2008 Paper 3 Question 6

## Logic and Proof

- (a) Draw the BDDs for the formulae  $(P \land Q) \to R$  and  $(P \lor Q) \to R$ , ordering the variables alphabetically. [2+2 marks]
- (b) Combine those BDDs to obtain the BDD for  $[(P \land Q) \to R] \leftrightarrow [(P \lor Q) \to R]$ . Briefly explain your working. [5 marks]
- (c) Use the DPLL method to determine whether or not the following set of formulae is consistent.

$$(Q \to R) \lor P$$
$$R \to (\neg P \lor Q)$$
$$(\neg P) \leftrightarrow Q$$
$$P \to R$$

[6 marks]

(d) Use the sequent calculus to determine whether or not the following sequent is valid:

$$\forall x [P(x) \lor Q(f(x))], \exists y \neg P(y) \Rightarrow \exists y Q(y)$$

[5 marks]