## 2008 Paper 3 Question 6

## Logic and Proof

(a) Draw the BDDs for the formulae $(P \wedge Q) \rightarrow R$ and $(P \vee Q) \rightarrow R$, ordering the variables alphabetically.
(b) Combine those BDDs to obtain the BDD for $[(P \wedge Q) \rightarrow R] \leftrightarrow[(P \vee Q) \rightarrow R]$. Briefly explain your working.
(c) Use the DPLL method to determine whether or not the following set of formulae is consistent.

$$
\begin{gathered}
(Q \rightarrow R) \vee P \\
R \rightarrow(\neg P \vee Q) \\
(\neg P) \leftrightarrow Q \\
P \rightarrow R
\end{gathered}
$$

(d) Use the sequent calculus to determine whether or not the following sequent is valid:

$$
\forall x[P(x) \vee Q(f(x))], \exists y \neg P(y) \Rightarrow \exists y Q(y)
$$

