2008 Paper 2 Question 5

Probability

(a) Suppose that X is a random variable whose value r is distributed Geometric(p). Write down the expression for the probability $\mathrm{P}(X = r)$. [3 marks]

(b) By using a suitable generating function or otherwise, show that the expectation $\mathrm{E}(X) = (1 - p)/p$. [5 marks]

The University Computing Service define a serious power outage as a power cut that lasts for longer than their Uninterruptable Power Supply equipment can maintain power. During the course of an academical year the number of serious power outages is a random variable whose value is distributed Geometric(2/5). Accordingly, the probability of having no serious power outages during the course of a year is 2/5.

(c) The University is investigating a compensation scheme which would make no payment over the year if the number of serious power outages were zero or one but which would pay the Computing Service £1000 for every such outage (including the first) if the total number of serious power outages in a year were two or more. Determine the expected annual sum that the Computing Service would receive. [8 marks]

(d) To what value would the parameter of the Geometric Distribution have to be changed (from 2/5) for the expected annual sum to be £750? [4 marks]