

## 2008 Paper 12 Question 10

### Computation Theory

(a) *The Halting Problem for register machines is unsolvable.* State, without proof, a precise form of this result. [3 marks]

(b) Let the computation by program  $c$  on data  $d$  be represented by the natural number  $k$  that codes the pair  $(c, d)$ . By considering the set  $H(k)$  of the HALTING computations represented by codes  $k' < k$ , show that there is an increasing total function  $h(k)$  which *grows too fast* to be computable. [6 marks]

(c) Given  $h : \mathbb{N} \rightarrow \mathbb{N}$  with the above property

$$\begin{aligned} \text{let } f(k) &= h(k) + k \\ \text{and } g(x) &= \sup\{k : f(k) \leq x\}. \end{aligned}$$

Then  $f : \mathbb{N} \rightarrow \mathbb{N}$  is strictly increasing, and  $g : \mathbb{N} \rightarrow \mathbb{N}$  satisfies

$$g(f(k)) = k, \quad g(x) < k \quad \text{for all } x < f(k).$$

Show that  $g$  *grows too slowly* to be computable in the following sense:

given  $G : \mathbb{N} \rightarrow \mathbb{N}$  such that

- (i)  $\{G(n) : n \in \mathbb{N}\}$  is unbounded
- (ii)  $G(n) \leq g(n)$  for all  $n \in \mathbb{N}$

then  $G(n)$  is *not* computable.

[11 marks]