

## 2007 Paper 8 Question 7

### Information Theory and Coding

- (a) Suppose that  $X$  is a random variable whose entropy  $H(X)$  is 8 bits. Suppose that  $Y(X)$  is a deterministic function that takes on a different value for each value of  $X$ .
- (i) What then is  $H(Y)$ , the entropy of  $Y$ ? [1 mark]
  - (ii) What is  $H(Y|X)$ , the conditional entropy of  $Y$  given  $X$ ? [1 mark]
  - (iii) What is  $H(X|Y)$ , the conditional entropy of  $X$  given  $Y$ ? [1 mark]
  - (iv) What is  $H(X, Y)$ , the joint entropy of  $X$  and  $Y$ ? [1 mark]
  - (v) Suppose now that the deterministic function  $Y(X)$  is not invertible; in other words, different values of  $X$  may correspond to the same value of  $Y(X)$ . In that case, what could you say about  $H(Y)$ ? [2 marks]
  - (vi) In that case, what could you say about  $H(X|Y)$ ? [1 mark]
- (b) A continuous real-valued signal has a bandwidth limited to  $\pm W$  Hertz.
- (i) In a duration of time  $T$ , at most how many regularly-spaced samples are needed in order for the signal to be specified completely at all points within  $T$ ? State the theorem that is the basis of your answer. [2 marks]
  - (ii) What about the signal values in between the points that are sampled – how can anything be known about those unobserved values? [1 mark]
- (c) Write down the general functional form for a 1-D Gabor wavelet, and explain how particular choices for the values of its parameters would turn it into either the Fourier basis or the delta function sampling basis, as two special cases. [3 marks]
- (d) Show that the set of all Gabor wavelets is closed under convolution. That is, show that the convolution of any two Gabor wavelets is also a Gabor wavelet. Comment on how this property relates to the fact that these wavelets are also closed under multiplication, and that they are also self-Fourier. [3 marks]
- (e) We wish to compute the Fourier Transform of a data sequence of 1,024 samples:
- (i) Approximately how many multiplications would be needed if the Fourier integral expressions were to be computed literally (as written mathematically) and without a clever algorithm? [2 marks]
  - (ii) Approximately how many multiplications would be needed if an FFT algorithm were used? [2 marks]