2007 Paper 7 Question 10

Computer Systems Modelling

(a) Let U be a uniform $(0, 1)$ random variable. Show that for any continuous distribution function $F(x)$, the random variable X defined by

$$X = F^{-1}(U)$$

has the probability distribution function $F(x)$. [4 marks]

(b) Use your result in part (a) and a uniform $(0, 1)$ random variable, U, to construct random variables for the following two distributions:

    (i)   the uniform (a, b) distribution where a and b are real numbers such that $a < b$; [3 marks]

    (ii) the exponential distribution $Exp(\lambda)$ with parameter $\lambda > 0$. [3 marks]

(c) Suppose that $X_1, X_2, \ldots, X_n$ are independent, identically distributed random variables with mean μ and variance σ^2. Use the central limit theorem to derive an approximate $100(1 - \alpha)$ percent confidence interval for μ. [5 marks]

(d) How would you obtain a confidence interval similar to that given in part (c) that is exact in the special case where the random variables $X_1, X_2, \ldots, X_n$ have a Normal distribution? [5 marks]