

2007 Paper 4 Question 6

Computation Theory

- (a) What does it mean for a set of natural numbers $S \subseteq \mathbb{N}$ to be
- (i) *recursive*? [1 mark]
 - (ii) *recursively enumerable*? [2 marks]
- (b) Show that if a set is recursive, then it is also recursively enumerable. [5 marks]
- (c) Let ϕ_e denote the partial function from \mathbb{N} to \mathbb{N} computed by the register machine with code $e \in \mathbb{N}$. Is either of the following sets of numbers recursively enumerable? Justify your answer in each case, stating clearly any standard results that you use.
- (i) $S_1 = \{e \in \mathbb{N} \mid \text{for all } x \in \mathbb{N}, \phi_e(x) \text{ is defined}\}$. [6 marks]
 - (ii) $S_2 = \{e \in \mathbb{N} \mid \text{for some } x \in \mathbb{N}, \phi_e(x) \text{ is defined}\}$. [6 marks]