2007 Paper 4 Question 5

Mathematical Methods for Computer Science

(*a*) Suppose that X_n is an irreducible Markov Chain with transition matrix P and suppose that π is a probability distribution over the states of the Markov Chain.

    (*i*) State the detailed balance conditions for the Markov Chain to be reversible in terms of the distribution π.   [4 marks]

    (*ii*) Show that if the detailed balance conditions hold then π is a stationary distribution for the Markov Chain.   [4 marks]

Suppose that G is a graph with vertices $i \in N$ and undirected edges $(i,j) \in E$ where N and E are both finite sets. Assume that G is connected so that there is a path between every pair of nodes. Define a Markov Chain on the graph G with states given by the vertices, N, and transition matrix P such that $P_{ij} = 1/v_i$ if $(i,j) \in E$ and $P_{ij} = 0$ otherwise where v_i is the number of edges incident at vertex i ($i \in N$).

(*b*) Show that $\pi_i = v_i / \sum_{j \in N} v_j$ for $i \in N$ is in detailed balance with P.   [6 marks]

(*c*) Suppose that G has eight vertices and undirected edges as shown in the figure:
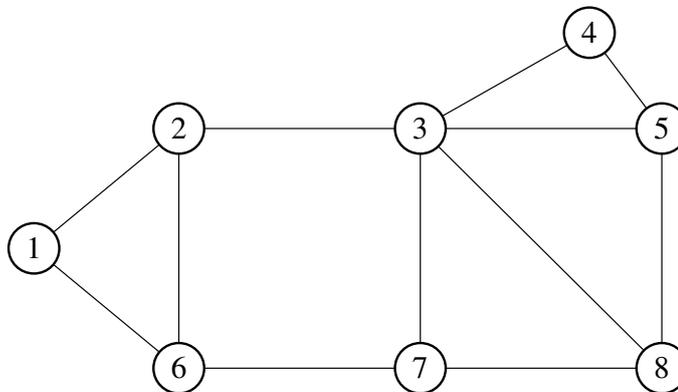
Find the stationary distribution for the Markov Chain on G and determine the relative proportions of time spent at each of the eight vertices. [6 marks]