

2007 Paper 3 Question 6

Mathematical Methods for Computer Science

Let

$$b_a(x) = \begin{cases} 1 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$$

where a is a constant such that $0 < a \leq \pi$.

- (a) Find the Fourier Transform, $F_a(\omega)$, of $b_a(x)$. [6 marks]
- (b) Suppose that $f(x)$ is some 2π -periodic function with complex Fourier coefficients, c_n , for $n = 0, \pm 1, \pm 2, \dots$
- (i) State an expression for c_n , for $n = 0, \pm 1, \pm 2, \dots$ [2 marks]
- (ii) Show that $c_n = G(n)$ for $n = 0, \pm 1, \pm 2, \dots$ where the function $G(\omega)$ is the Fourier transform of $f(x)b_\pi(x)$. [6 marks]
- (c) Now suppose that $f(x)$ is the 2π -periodic function defined such that $f(x) = b_a(x)$ for $|x| \leq \pi$. Find the complex Fourier coefficients, c_n , for this choice of the function $f(x)$ using your result derived in part (b)(ii). [6 marks]