2007 Paper 3 Question 2

Probability

(a) The notation $\binom{n}{r}$ may be interpreted as the number of ways of choosing r items from n. Give an elementary proof of Pascal's theorem that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$ given $n \geqslant 1$ and $0 < r < n$. The proof need not be very formal but do not exploit the representation that makes use of factorials. [4 marks]

(b) A College computer officer discovers that two of the workstations in a computer room have dirty keyboards. All the other keyboards are clean. No one else is in the room at the time of her inspection but she notices that four students enter the room just as she is leaving to fetch her cleaning materials.

If all four students keep clear of the dirty keyboards the computer officer won't have to disturb anyone on her return. Assuming that the students choose workstations at random she determines the probability a that all four happen to sit at clean keyboards. She also determines the probability b that both the workstations with dirty keyboards are amongst the four chosen. She is intrigued to note that $a = b$.

(i) Assuming that there are n workstations in the room, show that

$$a = \frac{n-4}{n} \cdot \frac{n-5}{n-1}$$ [4 marks]

(ii) Likewise express the probability b in terms of n. [10 marks]

(iii) By equating a and b determine the number of workstations in the computer room. [2 marks]