2007 Paper 10 Question 7

Computation Theory

(*a*) (*i*) Define the notion of a *register machine* and the computations that it carries out. [5 marks]

(*ii*) Explain, in general terms, what is meant by a *universal* register machine. (You should make clear what scheme for coding programs as numbers you are using, but you are not required to describe a universal register machine program in detail.) [5 marks]

(*b*) (*i*) Explain what it means for a partial function f from $\mathbb{N}$ to $\mathbb{N}$ to be *computable* by a register machine. [2 marks]

(*ii*) Let $n > 1$ be a fixed natural number. Show that the partial function from $\mathbb{N}$ to $\mathbb{N}$

$$f_n(x) = \begin{cases} nx & \text{if } x > 0 \\ \text{undefined} & \text{if } x = 0 \end{cases}$$

is computable. [3 marks]

(*iii*) Explain why there are only countably many computable functions from $\mathbb{N}$ to $\mathbb{N}$. Deduce that there exists a partial function from $\mathbb{N}$ to $\mathbb{N}$ that is not computable. (Any standard results you use about countable and uncountable sets should be clearly stated, but need not be proved.) [3 marks]

(*iv*) If a partial function f from $\mathbb{N}$ to $\mathbb{N}$ is computable, how many different register machine programs are there that compute f? [2 marks]