

2006 Paper 7 Question 8

Information Theory and Coding

- (a) Give *three* different expressions for mutual information $I(X;Y)$ between two discrete random variables X and Y , in terms of their two conditional entropies $H(X|Y)$ and $H(Y|X)$, their marginal entropies $H(X)$ and $H(Y)$, and their joint entropy $H(X,Y)$. Explain in ordinary language the concept signified by each of the measures $H(X|Y)$, $H(X)$, $H(X,Y)$, and $I(X;Y)$. Depict in a Venn diagram the relationships among all of the quantities mentioned here.

[8 marks]

- (b) Suppose that women who live beyond the age of 70 outnumber men in the same age bracket by three to one. How much information, in bits, is gained by learning that a certain person who lives beyond 70 happens to be male?

[2 marks]

- (c) What is the shortest possible code length, in bits per average symbol, that could be achieved for a six-letter alphabet whose symbols have the following probability distribution?

$$\left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32} \right\}$$

[3 marks]

- (d) If we wish to increase the transmission capacity of a noisy communication channel, is it more effective to increase its electronic bandwidth in Hertz, or to improve its signal-to-noise ratio? Briefly say why.

[2 marks]

- (e) A continuous signal whose total bandwidth is 1 kHz and whose duration is 10 seconds may be perfectly represented (even at points in between the points at which it is sampled) by what minimal number of real numbers?

[2 marks]

- (f) Give the names of *three* functions (not necessarily their equations) which are self-Fourier.

[3 marks]