2006 Paper 3 Question 6

Numerical Analysis I

(*a*) An IEEE Single Precision number is stored in 32 bits, of which 8 bits are reserved for the exponent. Explain the terms *normalised number* and *hidden bit*. How many bits are used to store the *significand*, and what is the *precision*? Show by means of a diagram how the bits are arranged in storage. [4 marks]

(*b*) How is the value of the exponent stored? What are the stored values of the exponents e_{min} and e_{max}? [3 marks]

(*c*) Which values are represented by the following bit patterns? [Show signs where appropriate.]

(*i*) 00000000 00000000 00000000 00000000

(*ii*) 11111111 11111111 11111111 11111111

(*iii*) 00111111 10000000 00000000 00000000

(*iv*) 11000000 00000000 00000000 00000000

(*v*) 00000000 00000000 00000000 00000001

(*vi*) 01111111 10000000 00000000 00000000 [6 marks]

(*d*) Define *machine epsilon* ε_m. Estimate its value in IEEE Single Precision. [2 marks]

(*e*) What are the two sources of error in the formula

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}$$

and how does each type of error behave as h increases? Suggest a suitable value of h to use with this formula for IEEE Single Precision when $f(x) = O(1)$. [5 marks]