Databases

- (a) Define Boyce–Codd normal form. [3 marks]
- (b) Suppose that a relation R has n attributes. How many distinct functional dependencies could be defined for R? [3 marks]
- (c) The union rule for functional dependencies states that if $F \models X \to Y$ and $F \models X \to Z$, then $F \models X \to Y \cup Z$ (this can also be written as $F \models X \to Y, Z$).

Prove this rule using only Armstrong's axioms. [5 marks]

(d) Heath's Theorem states that if R(A, B, C) satisfies the functional dependency $A \rightarrow B$, where A, B, and C are disjoint non-empty sets of attributes, then

$$R = \pi_{A,B}(R) \bowtie_A \pi_{A,C}(R),$$

where \bowtie_A is the equi-join on the attributes of A. Prove this theorem.

[9 marks]