2006 Paper 11 Question 9

Computation Theory

(*a*) Define the collection of *primitive recursive* functions. [6 marks]

(*b*) Why is a primitive recursive function always total? [1 mark]

(*c*) Show that the function m from $\mathbb{N}^2$ to $\mathbb{N}$ given by

$$m(x, y) = \begin{cases} x - y & \text{if } x \geq y \\ 0 & \text{if } x < y \end{cases}$$

is primitive recursive. [3 marks]

(*d*) Define the collection of *partial recursive* functions. [3 marks]

(*e*) What is meant by a *total recursive* function? [1 mark]

(*f*) Show that there exist total recursive functions that are not primitive recursive. Any standard results about register machines or recursive functions that you use need not be proved, but should be clearly stated. [6 marks]