2005 Paper 9 Question 15

Denotational Semantics

(*a*) Describe how to construct the function cpo $((D \to E), \sqsubseteq)$ of two cpos $(D, \sqsubseteq_D)$ and $(E, \sqsubseteq_E)$. Prove that $((D \to E), \sqsubseteq)$ is a cpo.
(You may use general facts about least upper bounds provided you state them clearly.) [7 marks]

(*b*) The function *uncurry* is inverse to the function *curry*; it takes a continuous function in $(D_1 \to (D_2 \to E))$ as argument and yields a continuous function in $((D_1 \times D_2) \to E)$ as result. Give a definition of *uncurry* and show it is a continuous function.
(You may use general facts about continuous functions provided you state them clearly.) [6 marks]

(*c*) Exhibit two terms of PCF which are contextually equivalent and yet have distinct denotations in the domain $(\mathbb{B}_\perp \to (\mathbb{B}_\perp \to \mathbb{B}_\perp)) \to \mathbb{B}_\perp$ where $\mathbb{B} = \{\mathit{true}, \mathit{false}\}$ is the set of truth values. Explain why their denotations differ. [7 marks]