2005 Paper 4 Question 7

Numerical Analysis I

(a) Define *absolute error* and *relative error*. How are these related? Explain briefly the term *loss of significance*. [3 marks]

(b) An algorithm is required for solution of $ax^2 + bx + c = 0$ where $b > 0$. Describe how loss of significance can occur in the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and derive an alternative formula for such a case. Illustrate your answer by applying it to the case $a = 30$, $b = 3000$, $c = 1$ on a decimal machine with only 5 significant digits available. [You should assume for the purposes of calculation that $\sqrt{b^2 - 4ac}$ evaluates to $b - (2ac/b)$, correctly rounded.] [10 marks]

(c) The series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

is to be summed by taking terms in order, left to right, using only p decimal digits of precision until additional terms are negligible. If $x = 6$ find the largest term of the series and hence, assuming $\cos 6 \simeq 1$, estimate roughly how many decimal digits of accuracy will be lost in the process. [7 marks]