2005 Paper 2 Question 4

Probability

(*a*) Given a random variable X distributed Geometric(p) one has:

$$\mathrm{P}(X = r) = q^r p \quad \text{where} \quad p + q = 1$$

By using a suitable generating function, derive expressions for the expectation and variance of X. [6 marks]

(*b*) A computer game proceeds in a sequence of steps and, at each step, the contents of a two-bit random number generator are inspected. Each bit is 1 with probability p and 0 with probability q where $p + q = 1$. The two bits are independent.

If the two bits differ (as 01 or 10) the game ends but if the two bits are the same (as 00 or 11) the game proceeds to another step. Accordingly, the game could be as short as a single step but it could last many steps.

Let X be a random variable whose value r is the length of a game measured in steps. The game cannot end at step 0 so $\mathrm{P}(X = 0) = 0$.

(*i*) Derive an expression for $\mathrm{P}(X = r)$. [5 marks]

(*ii*) Show that $\sum_{r=0}^{\infty} \mathrm{P}(X = r) = 1$. [4 marks]

(*iii*) Derive an expression for the expected length of a game measured in steps. [5 marks]