2005 Paper 12 Question 1

Data Structures and Algorithms

(*a*) Explain how a Boolean matrix can be used to represent the edges of a finite directed graph whose vertices are numbered 1 to n. [2 marks]

(*b*) Describe Warshall's algorithm to convert the matrix representing a graph to one that represents its transitive closure, and carefully explain why the algorithm works. [6 marks]

(*c*) Outline Floyd's algorithm, without proof of correctness, to find the cost of the cheapest path between any two vertices of a directed graph where the edges carry non-negative costs. [4 marks]

(*d*) It is required to construct a matrix R that encodes a path with the minimum number of edges from any vertex i to any other vertex j. R_{ij} will be zero if no path exists from vertex i to vertex j; otherwise, R_{ij} will hold the vertex number of the next vertex of a minimal path from i to j. Suggest an algorithm to compute R from a given Boolean matrix M. [8 marks]