# 2005 Paper 11 Question 9

**Mathematics for Computation Theory**

(*a*) Let $M$ be an $n$-state deterministic finite automaton over the finite alphabet $S$. Write $l(w)$ for the length of words $w \in S^*$. Suppose that $M$ accepts the word $x \in S^*$, where $l(x) \geqslant n$.

(*i*) Show that $x$ is a concatenation of words $uvw$, where $l(uv) \leqslant n$, $l(v) \geqslant 1$, and $M$ accepts the word $z_k = uv^k w$ for all natural numbers $k \geqslant 0$. [8 marks]

(*ii*) Hence show that if $M$ accepts some word $y \in S^*$, it must accept some word $z \in S^*$ such that $l(z) < n$; and that $M$ accepts an infinite set of words if and only if it accepts some word $x \in S^*$ such that $n \leqslant l(x) < 2n$. [5 marks]

(*b*) Let $S = \{a, b\}$ be an alphabet of two symbols. Explain whether each of the following languages over $S$ is regular:

(*i*) $L_1 = \{uv \mid u, v \in S^*, \quad l(v) = 2.l(u)\}$ [3 marks]

(*ii*) $L_2 = \{ww \mid w \in S^*\}$ [4 marks]