COMPUTER SCIENCE TRIPOS Part II (General)
DIPLOMA IN COMPUTER SCIENCE

Monday 6 June 2005 1.30 to 4.30

Paper 10 (Paper 1 of Diploma in Computer Science)

Answer **five** *questions.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script Paper
Blue Coversheets
Tags

1 Digital Electronics

(*a*) What is a *minimum sum-of-products*? [3 marks]

(*b*) A full adder has data inputs (A_0, B_0) and a carry input (C_0). The sum (S_0) and carry (C_1) are output. What are the minimum sum-of-products equations for S_0 and C_1? [6 marks]

(*c*) How could the gate count for the implementation of output S_0 be reduced using XOR gates? [2 marks]

(*d*) For a 3-bit full adder (i.e. one which has three A inputs (A_0, A_1, A_2), three B inputs (B_0, B_1, B_2) and three sum outputs (S_0, S_1, S_2)), the final carry output is C_3. What is the sum-of-products equation for C_3 in terms of the A and B inputs? [6 marks]

(*e*) If we were to implement an 8-bit full adder, why would we look for a multi-level logic implementation for the carry output (C_8)? [3 marks]

2 Foundations of Programming

(a) Distinguish between the terms *instance method* and *class method*. [4 marks]

(b) A newcomer to Java programming has written the following code:

```
class Parent
 { public void test()
    { System.out.println("Parent");
    }
 }

public class Child extends Parent
 { public static void main(String[] args)
    { Parent p = new Parent();
      Child c = new Child();
      p.test();
      c.test();

      p = c;
      p.test();
      c.test();

      c = p;
      p.test();
      c.test();
    }

   public void test()
    { System.out.println("Child");
    }
 }
```

The `javac` compiler complains about one statement. Which one and why? Correct the code by inserting an appropriate cast. [4 marks]

(c) With this correction the program will compile and run. Explain in outline what happens at run-time and show what output is printed. [5 marks]

(d) Small print in the Java documentation says that you "cannot override a `static` method but you can hide it". If both `test()` methods are made `static` the program will again compile and run. Explain what happens this time and show what output is printed. [7 marks]

3 Data Structures and Algorithms

(*a*) Briefly outline how a sequence of symbols can be encoded as a sequence of Huffman codes, and explain under what assumptions Huffman encoding generates optimally compact code. [6 marks]

(*b*) Estimate the number of bits needed to Huffman encode a random permutation of `A`s, `B`s and `C`s, with each letter occurring one million times. [3 marks]

(*c*) Estimate the number of bits needed to Huffman encode a random permutation of `A`s, `B`s and `C`s, where `A` occurs two million times and `B` and `C` each occur one million times. [3 marks]

(*d*) Estimate how many bits would be needed to encode the sequence in part (*b*) above using arithmetic coding. You may assume that $\log_2 3$ is about 1.6. [4 marks]

(*e*) Estimate, with justification, how many bits would be needed to encode the sequence in part (*c*) above using arithmetic coding. [4 marks]

4 Artificial Intelligence I

(*a*) What are the advantages and disadvantages of *constraint satisfaction problem* (CSP) solvers compared with search algorithms such as $A^\star$ search, etc? [3 marks]

(*b*) Give a general definition of a CSP. Define the way in which a solution is represented and what it means for a solution to be *consistent* and *complete*. [5 marks]

(*c*) Assuming discrete binary constraints and finite domains, explain how breadth-first-search might be used to find a solution and why this is an undesirable approach. [3 marks]

(*d*) Give a brief description of the basic *backtracking algorithm* for finding a solution. [4 marks]

(*e*) Describe the *minimum remaining values heuristic*, the *degree heuristic* and the *least constraining value heuristic*. [5 marks]

5 Comparative Programming Languages

Consider the Prolog procedures named `s` and `p` defined as follows:

```
s(H, [H|T], T).
s(H, [N|T], [N|L]) :- s(H, T, L).

p(X, [H|T]) :- s(H, X, Z), p(Z, T).
p([], []).
```

(*a*) Show how Prolog would evaluate the goal `s(H, [a,b,c], T)` giving all the successive instantiations of `H` and `T` that cause the goal to be satisfied, and hence describe in words what `s` does. [6 marks]

(*b*) What value of `Q` causes the goal `p([a], Q)` to be satisfied? [3 marks]

(*c*) What values of `Q` cause the goal `p([a,b], Q)` to be satisfied? [4 marks]

(*d*) What values of `Q` cause the goal `p([a,b,c], Q)` to be satisfied? [5 marks]

(*e*) Describe in words what `p` does. [2 marks]

6 Operating System Foundations

(*a*) A device driver process carries out character I/O via a Universal Asynchronous Receiver/Transmitter (UART).

(*i*) Why is hardware–software synchronisation needed? [1 mark]

(*ii*) Describe polled operation. [2 marks]

(*iii*) Describe interrupt-driven operation. [2 marks]

(*iv*) Draw a state transition diagram for the device-driver process. Indicate the events that cause each transition and in each case explain the effect on the device driver's process descriptor and the operating system's scheduling queues. Assume interrupt-driven software. [7 marks]

(*b*) The device driver process fills/empties a buffer of fixed size in an I/O buffer area. A process carrying out application requests reads and writes data in variable-sized amounts from the buffer.

(*i*) Why must mutually exclusive access to the buffer be enforced? [2 marks]

(*ii*) Why is condition synchronisation needed? [2 marks]

(*iii*) What is wrong with the following pseudocode fragment from the device-driver's specification, where:

- `buffer-lock` is a semaphore initialised to `1`,
- `space` is a semaphore initialised to the buffer size in bytes,
- `data` is a semaphore initialised to `0`?

on input:	on output:
`WAIT(buffer-lock);`	`WAIT(buffer-lock);`
`if` buffer is full `then WAIT(space);`	`if` buffer is empty `then WAIT(data);`
write a character into the buffer;	read a character from the buffer;
`SIGNAL(buffer-lock);`	`SIGNAL(buffer-lock);`

[4 marks]

7 Numerical Analysis I

(*a*) The parameters for *IEEE* Single Precision are: $\beta = 2$, $p = 24$, $e_{min} = -126$, $e_{max} = 127$. Explain the terms *significand*, *sign bit*, *exponent*, *normalised number*, *denormal number*, *hidden bit*, *precision* as used in *IEEE* Single Precision. [7 marks]

(*b*) Let ω represent any of the operations $+ \ - \ * \ /$. Let x be a positive finite representable number. List what each of the following evaluates to for each operation:

$$(+\infty) \ \omega \ x$$
$$x \ \omega \ (-\infty)$$

[Show the sign of your answer in each case.] [4 marks]

(*c*) Suppose the principles of *IEEE* arithmetic are applied to a floating-point representation with 6 bytes (48 stored bits). If $\beta = 2$, $e_{max} = 511$ and a hidden bit is used, deduce the values of e_{min} and p. [4 marks]

(*d*) Define *machine epsilon* ε_m. [1 mark]

(*e*) The function

$$f(x) = \frac{(x+1)^2}{x^2+1}$$

is to be evaluated using *IEEE* arithmetic for $x \geq 0$. Re-write the formula so that $f(x)$ can be evaluated in the case where x is representable but x^2 overflows. What does your formula evaluate to in the case that $(1/x) < \varepsilon_m$? [4 marks]

8 Mathematics for Computation Theory

State the requirements for $(S, \leqslant)$ to be:

(a) a *partially ordered* set;

(b) a *totally ordered* set;

(c) a *well ordered* set. [5 marks]

Let $(\mathbb{N}, \leqslant)$ be the natural numbers under the standard ordering. Define the *product ordering* $\leqslant_p$ on $(\mathbb{N} \times \mathbb{N})$ that is derived from this ordering. Which of conditions (a), (b), (c) does $\leqslant_p$ satisfy? [3 marks]

Let $(S, \leqslant)$ and $(T, \prec)$ be partially ordered sets, and $f : (S, \leqslant) \to (T, \prec)$ be a function. What condition must be satisfied in order that f be *monotonic*? [2 marks]

If f is a bijection, and both f and f^{-1} are monotonic, we say that $(S, \leqslant), (T, \prec)$ are *isomorphic* partially ordered sets.

Suppose that $(S, \leqslant)$ is a partially ordered set. A *topological sort* of $(S, \leqslant)$ is defined by specifying a total ordering $\sqsubseteq$ on S such that the identity map $\imath : (S, \leqslant) \to (S, \sqsubseteq)$ is monotonic.

Define *two* different topological sorts of $(\mathbb{N} \times \mathbb{N}, \leqslant_p)$, one of which is isomorphic to $\mathbb{N}$ with the standard ordering, while the other is not. Justify your claims. [10 marks]

9 Computation Theory

(a) Explain informally, i.e. without reference to any particular model of computation, why the *Halting Problem* is undecidable. [6 marks]

(b) Briefly describe two mathematical problems, other than the Halting Problem, that are algorithmically undecidable. [4 marks]

(c) What does it mean for a partial function to be *register machine computable*? Show how the informal argument in part (a) can be turned into a rigorous proof that there is no register machine deciding the Halting Problem for register machine computable functions. [10 marks]

10 Computer Graphics and Image Processing

(*a*) Calculate the maximum resolution needed by a movie projector in a movie theatre. Clearly state any assumptions that you make. [6 marks]

(*b*) Describe, in detail, an error diffusion algorithm for converting greyscale images to bi-level black and white images at the same resolution. [8 marks]

(*c*) Explain how this could be extended to provide an algorithm to print full colour RGB images on a CMYK laser printer, assuming that one pixel in the image maps to one pixel on the printer. [6 marks]

11 Introduction to Security

(*a*) A and B play a simple game. A chooses a number $R_A \in \mathbb{Z}_3$ and B chooses a number $R_B \in \mathbb{Z}_3$. Then A and B communicate their respective choice to each other *simultaneously*, meaning that the players cannot change their choice after having seen that of the opponent. These rules decide who wins the game:

$$\begin{aligned} R_A &\equiv R_B + 1 \pmod 3 \quad \Rightarrow \quad A \text{ wins} \\ R_B &\equiv R_A + 1 \pmod 3 \quad \Rightarrow \quad B \text{ wins} \end{aligned}$$

In any other case, the result of the game is a draw.

(*i*) What complication arises when this game is played at a distance, for example via e-mail? [2 marks]

(*ii*) Suggest a cryptographic protocol that prevents cheating when this game is played via e-mail. Your solution should not rely on a trusted third party. [6 marks]

(*iii*) What assumptions do you make about the cryptographic functions used in your solution of part (*ii*)? [3 marks]

(*iv*) What assumptions do you make about the amount of computing power available to the opponent in your solution of part (*ii*)? [3 marks]

(*b*) Outline briefly the purpose of an organisation's security policy and what steps should be considered in its development. [6 marks]

12 Software Engineering and Design

Imagine that you are the user interface designer responsible for a system that manages the shutdown of a nuclear power station.

(*a*) Comment on hazards, risks and reliability. [3 marks]

(*b*) What special procedures should be followed during design? [3 marks]

(*c*) There is some debate among the team about whether the operator should be in the control loop. What options are there? [4 marks]

(*d*) In order to assess alternative options like those in part (*c*):

(*i*) How could you *estimate* the speed of operator action based on a draft interface layout? [4 marks]

(*ii*) How could you *measure* the speed of operator action using alternative prototypes? [4 marks]

(*iii*) How could you estimate the probability of operator error? [2 marks]

END OF PAPER