2004 Paper 8 Question 10

Information Theory and Coding

(a) Consider a binary symmetric communication channel, whose input source is the alphabet $X = \{0, 1\}$ with probabilities $\{0.5, 0.5\}$; whose output alphabet is $Y = \{0, 1\}$; and whose channel matrix is

$$\begin{pmatrix} 1 - \epsilon & \epsilon \\ \epsilon & 1 - \epsilon \end{pmatrix}$$

where ϵ is the probability of transmission error.

   (i)   What is the entropy of the source, $H(X)$?   [2 marks]

   (ii)   What is the probability distribution of the outputs, $p(Y)$, and the entropy of this output distribution, $H(Y)$?   [2 marks]

   (iii) What is the joint probability distribution for the source and the output, $p(X, Y)$, and what is the joint entropy, $H(X, Y)$?   [2 marks]

   (iv) What is the mutual information of this channel, $I(X; Y)$?   [2 marks]

   (v)   How many values are there for ϵ for which the mutual information of this channel is maximal? What are those values, and what then is the capacity of such a channel in bits?   [2 marks]

   (vi) For what value of ϵ is the capacity of this channel minimal? What is the channel capacity in that case?   [2 marks]

(b) A variable length, uniquely decodable code which has the prefix property, and whose N binary code word lengths are $n_1 \leq n_2 \leq n_3 \leq \cdots \leq n_N$ must satisfy what condition on code word lengths? (State the condition, and name it.)
      [3 marks]

(c) You are asked to compress a collection of files, each of which contains several thousand photographic images. All images in a single file show the same scene. Everything in this scene is static (no motion, same camera position, etc.) except for the intensity of the five light sources that illuminate everything. The intensity of each of the five light sources changes in completely unpredictable and uncorrelated ways from image to image. The intensity of each pixel across all photos in a file can be described as a linear combination of the intensity of these five light sources.

   (i)   Which one of the five techniques *discrete cosine transform*, *μ-law coding*, *2-D Gabor transform*, *Karhunen-Loève transform* and *Golomb coding* would be best suited to remove redundancy from these files, assuming your computer is powerful enough for each?   [1 mark]

   (ii) Explain briefly this transform and why it is of use here.   [4 marks]